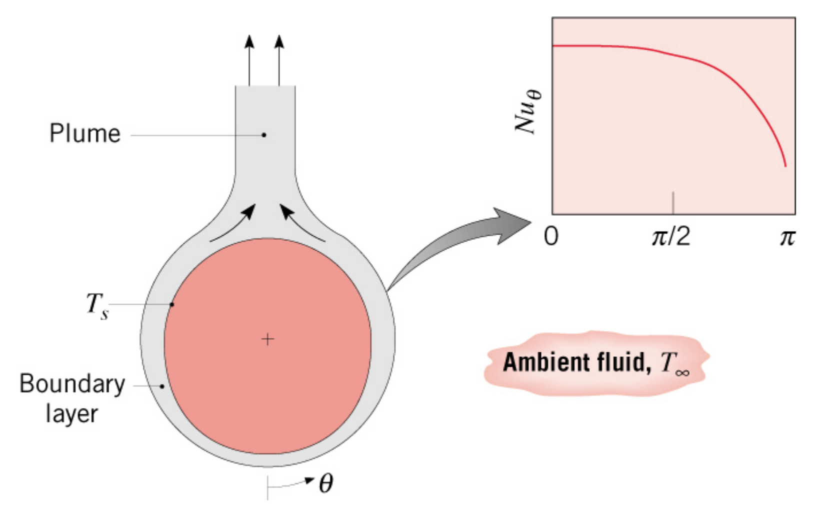
Free Convection:
Cylinders, Spheres, and Enclosures

Chapter 9

Section 9.6.3 through 9.8

The Long Horizontal Cylinder

- Boundary Layer Development and Variation of the Local Nusselt Number for a Heated Cylinder:



- The Average Nusselt Number:

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/\Pr)^{9/16}\right]^{8/27}} \right\}^2 \qquad Ra_D < 10^{12}$$

- How do conditions change for a cooled cylinder?

Spheres

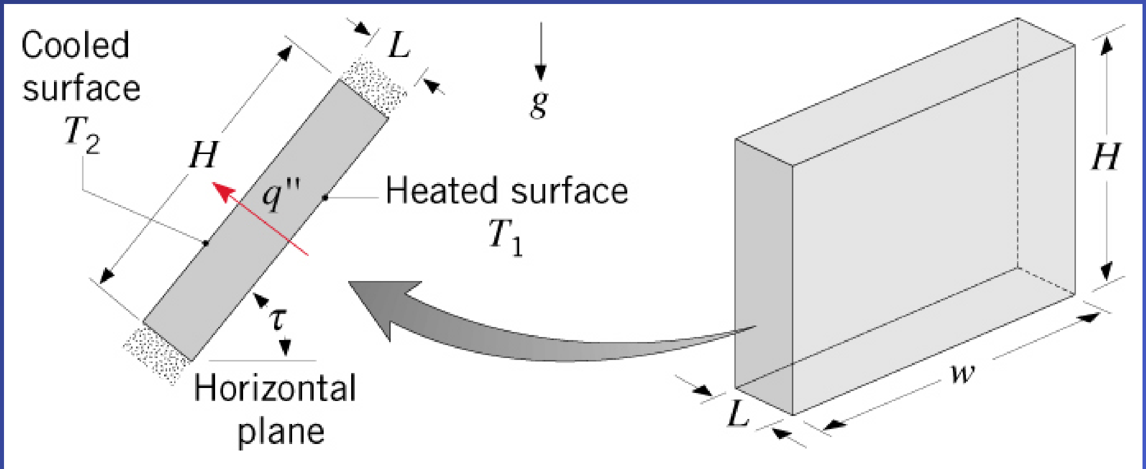
- The Average Nusselt Number:

$$\overline{Nu}_D = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1+\left(0.469/\Pr\right)^{9/16}\right]^{4/9}}$$

  - In the limit as $Ra_D \rightarrow 0$, how may conditions be characterized?

Enclosures

- Rectangular Cavities



- Characterized by opposing walls of different temperatures, with the remaining walls well insulated.

- $$Ra_L \equiv \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu}$$

  $$q'' = h(T_1 - T_2)$$

- Horizontal Cavity $\rightarrow \tau = 0, 180\,\text{deg}$

- Vertical Cavity $\rightarrow \tau = 90$ deg

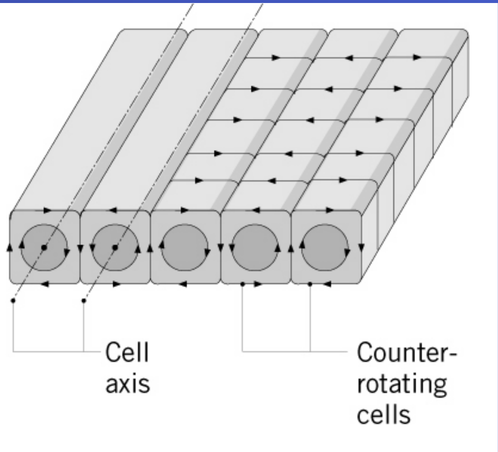
- Horizontal Cavities
  - Heating from Below $(\tau = 0)$
    - $Ra_L < Ra_{L,c} = 1708$:

      Fluid layer is thermally stable.

      $$\overline{Nu}_L = \frac{\bar{h}L}{k} = 1$$

    - $1708 < \mathrm{Ra}_L < 5 \times 10^4$:

      Thermal instability yields a regular convection pattern in the form of roll cells.



    - $3 \times 10^5 < Ra_L < 7 \times 10^9$:

      Buoyancy driven flow is turbulent

      $$\overline{Nu}_L = 0.069\ Ra_L^{1/3}\,\mathrm{Pr}^{0.074}$$

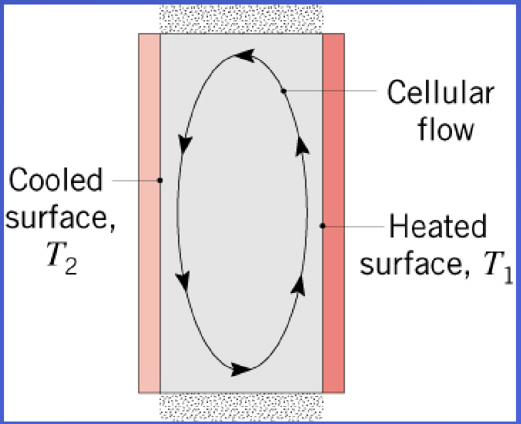
- Heating from Above $(\tau = 180 \text{ deg})$
  - Fluid layer is unconditionally stable.

$$\overline{Nu}_L = 1$$

- Vertical Cavities
  - $Ra_L < 10^3$:

$$\overline{Nu}_L = 1$$

  - $Ra_L > 10^3$:
    - A primary cellular flow is established, as the core becomes progressively more quiescent, and secondary (corner) cells develop with increasing Ra_L.

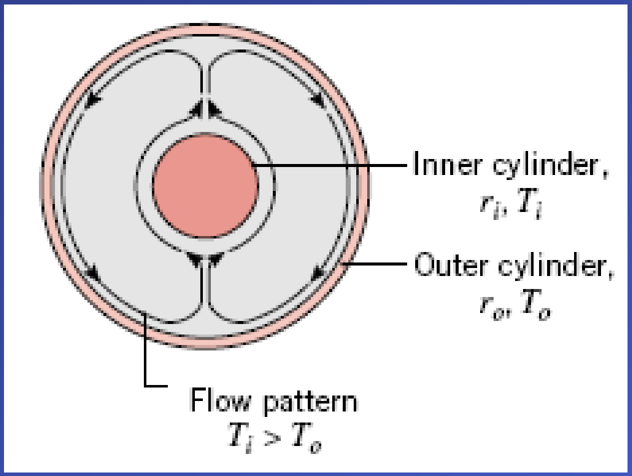


- Correlations for $\overline{Nu}_L \rightarrow$ Eqs. (9.50) - (9.53).

- Inclined Cavities
  - Relevant to flat plate solar collectors.
  - Heat transfer depends on the magnitude of τ relative to a critical angle τ^{*}, whose value depends on *H/L* (Table 9.4).
  - Heat transfer also depends on the magnitude of Ra_L relative to a critical Rayleigh number of $Ra_{L,c} = 1708/\cos\tau$.
  - Heat transfer correlations $\longrightarrow$ Eqs. (9.54) – (9.57).

Annular Cavities

- Concentric Cylinders



- $$q' = \frac{2\pi k_{eff}}{\ln(r_o/r_i)}(T_i - T_o)$$
- $$\frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4} Ra_c^{1/4}$$

or $k_{eff}/k = 1$ if the value calculated above is less than unity.

- The length scale in Ra_c is given by

$$L_c = \frac{2\left[ln\left(r_o / r_i\right)\right]^{4/3}}{\left(r_i^{-3/5} + r_o^{-3/5}\right)^{5/3}}$$

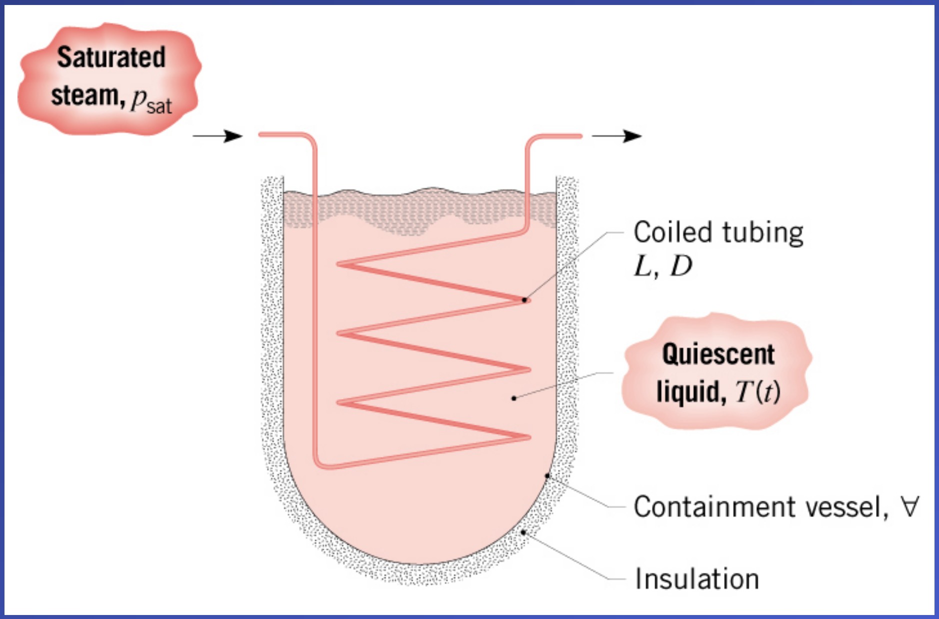
- Concentric Spheres
  - $$q = \frac{4\pi k_{eff}\left(T_i - T_o\right)}{\left(1/r_i\right) - \left(1/r_o\right)}$$
  - $$\frac{k_{eff}}{k} = 0.74\left(\frac{Pr}{0.861 + Pr}\right)^{1/4} Ra_s^{1/4}$$

  or $k_{eff}/k = 1$ if the value calculated above is less than unity.

  - The length scale in Ra_s is given by

$$L_s = \frac{\left(1/r_i - 1/r_o\right)^{4/3}}{2^{1/3}\left(r_i^{-7/5} + r_o^{-7/5}\right)^{5/3}}$$

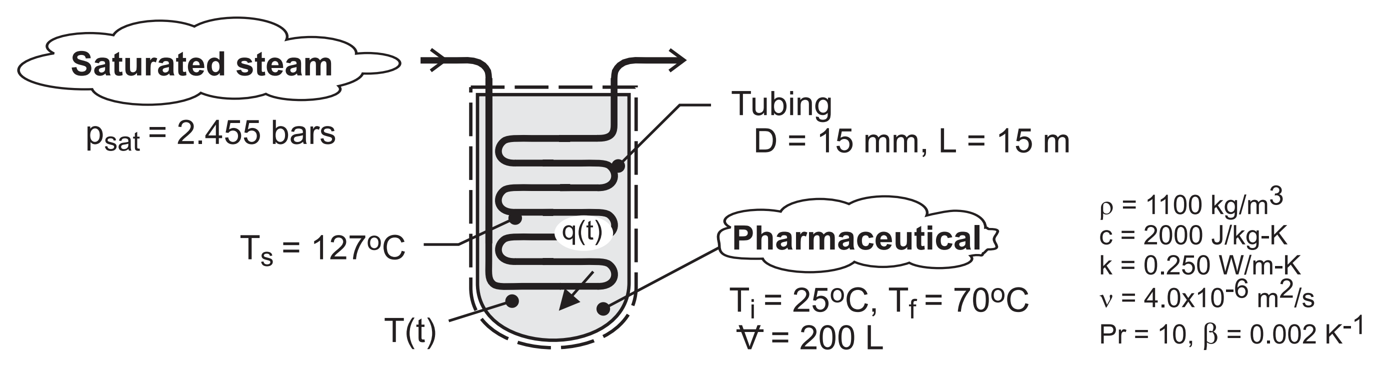
Problem 9.74: Use of saturated steam to heat a pharmaceutical in a batch reactor.



KNOWN: Volume, thermophysical properties, and initial and final temperatures of a pharmaceutical. Diameter and length of submerged tubing. Pressure of saturated steam flowing through the tubing.

FIND: (a) Initial rate of heat transfer to the pharmaceutical, (b) Time required to heat the pharmaceutical to 70°C and the amount of steam condensed during the process.

SCHEMATIC:



ASSUMPTIONS: (1) Pharmaceutical may be approximated as an infinite, quiescent fluid of uniform, but time-varying temperature, (2) Free convection heat transfer from the coil may be approximated as that from a heated, horizontal cylinder, (3) Negligible thermal resistance of condensing steam and tube wall, (4) Negligible heat transfer from tank to surroundings, (5) Constant properties.

PROPERTIES: *Table A-4*, Saturated water (2.455 bars): T_{sat} = 400K = 127°C, $h_{fg} = 2.183 \times 10^6$ J/kg. Pharmaceutical: See schematic.

ANALYSIS: (a) The initial rate of heat transfer is $q = \bar{h} A_s \left(T_s - T_i\right)$, where $A_s = \pi DL = 0.707$ m^2 and $\bar{h}$ is obtained from Eq. 9.34.

With $\alpha = \nu/\mathrm{Pr} = 4.0 \times 10^{-7}\ \mathrm{m^2/s}$ and $Ra_D = g\beta(T_s - T_i)\, D^3/\alpha\nu = 9.8\ \mathrm{m/s^2}\ (0.002\ \mathrm{K^{-1}})\ (102\mathrm{K})\ (0.015\mathrm{m})^3/16 \times 10^{-13}\ \mathrm{m^4/s^2} = 4.22 \times 10^6$,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(4.22 \times 10^6\right)^{1/6}}{\left[1 + (0.559/10)^{9/16}\right]^{8/27}}\right\}^2 = 27.7$$

Hence, $\quad \bar{h} = \overline{Nu}_D\, k/D = 27.7 \times 0.250\, \mathrm{W/m \cdot K}/0.015\mathrm{m} = 462\, \mathrm{W/m^2 \cdot K}$

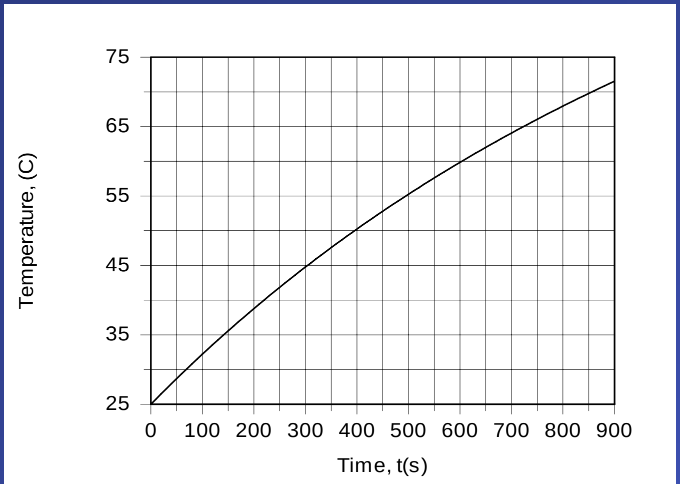
and $\quad q = \bar{h} A_s (T_s - T_i) = 462\, \mathrm{W/m^2 \cdot K} \times 0.707\, \mathrm{m^2}\, (102°\mathrm{C}) = 33{,}300\, \mathrm{W}$

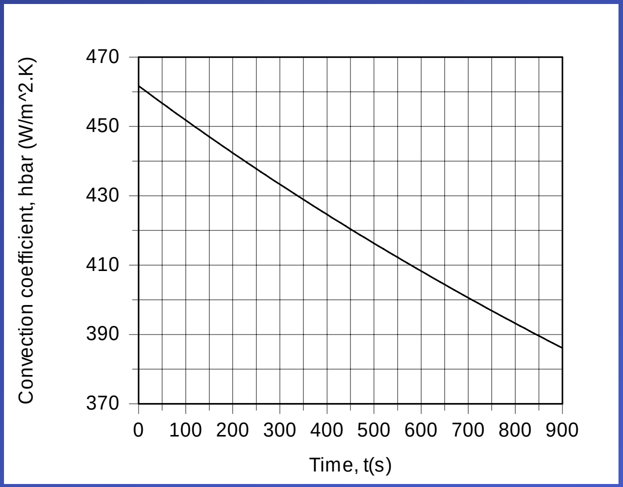
(b) Performing an energy balance at an instant of time for a control surface about the liquid,

$$\frac{d(\rho \forall c T)}{dt} = q(t) = \bar{h}(t) A_s \left(T_s - T(t)\right)$$

where the Rayleigh number, and hence $\bar{h}$, changes with time due to the change in the temperature of the liquid.

Integrating the foregoing equation numerically, the following results are obtained for the variation of T and $\bar{h}$ with t.





The time at which the liquid reaches 70°C is

$$t_f \approx 855\,s$$ <

The rate at which T increases decreases with increasing time due to the corresponding reduction in $(T_s - T)$, and hence reductions in Ra_D, $\bar{h}$ and q.

The Rayleigh number decreases from 4.22×10^6 to 2.16×10^6, while the heat rate decreases from 33,300 to 14,000 W.

The convection coefficient decreases approximately as $(T_s - T)^{1/3}$, while $q \sim (T_s - T)^{4/3}$.

The latent energy released by the condensed steam corresponds to the increase in thermal energy of the pharmaceutical. Hence, $m_c h_{fg} = \rho\forall c\left(T_f - T_i\right)$,

and

$$m_c = \frac{\rho\forall c\left(T_f - T_i\right)}{h_{fg}} = \frac{1100\,kg/m^3 \times 0.2\,m^3 \times 2000\,J/kg \cdot K \times 45°C}{2.183\times10^6\,J/kg} = 9.07\,kg \qquad <$$

COMMENTS: (1) Over such a large temperature range, the fluid properties are likely to vary significantly, particularly ν and Pr. A more accurate solution could therefore be performed if the temperature dependence of the properties were known. (2) Condensation of the steam is a significant process expense, which is linked to the equipment (capital) and energy (operating) costs associated with steam production.