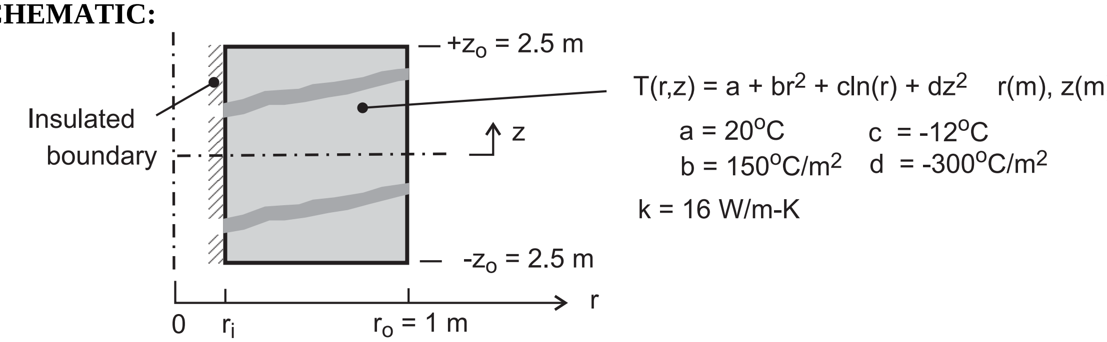
PROBLEM 2.40

KNOWN: Steady-state temperature distribution for hollow cylindrical solid with volumetric heat generation.

FIND: (a) Determine the inner radius of the cylinder, r_i, (b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}$, (c) Determine the axial distribution of the heat flux at the outer surface, $q''_r(r_o, z)$, and the heat rate at this outer surface; is the heat rate *in* or *out* of the cylinder; (d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q''_z(r, +z_o)$ and $q''_z(r, -z_o)$, and the corresponding heat rates; are the heat rates *in* or *out* of the cylinder; (e) Determine the relationship of the surface heat rates to the heat generation rate; is an overall energy balance satisfied?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction with constant properties and volumetric heat generation.

ANALYSIS: (a) Since the inner boundary, $r = r_i$, is adiabatic, then $q''_r(r_i, z) = 0$. Hence the temperature gradient in the r-direction must be zero.

$$\left.\frac{\partial T}{\partial r}\right)_{r_i} = 0 + 2br_i + c/r_i + 0 = 0$$

$$r_i = +\left(-\frac{c}{2b}\right)^{1/2} = \left(-\frac{-12°C}{2 \times 150°C/m^2}\right)^{1/2} = 0.2 \text{ m} \qquad <$$

(b) To determine $\dot{q}$, substitute the temperature distribution into the heat diffusion equation, Eq. 2.24, for two-dimensional (r,z), steady-state conduction

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\left[0 + 2br + c/r + 0\right]\right) + \frac{\partial}{\partial z}\left(0 + 0 + 0 + 2dz\right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r}\left[4br + 0\right] + 2d + \frac{\dot{q}}{k} = 0$$

$$\dot{q} = -k\left[4b + 2d\right] = -16\,\text{W/m}\cdot\text{K}\left[4 \times 150°\text{C/m}^2 + 2\left(-300°\text{C/m}^2\right)\right] = 0\,\text{W/m}^3 \qquad <$$

(c) The heat flux and the heat rate at the outer surface, $r = r_o$, may be calculated using Fourier's law.

$$q''_r(r_o, z) = -k\left.\frac{\partial T}{\partial r}\right)_{r_o} = -k\left[0 + 2br_o + c/r_o + 0\right]$$

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PROBLEM 2.40 (Cont.)

$$q''_r(r_{o},z) = -16\ \text{W/m}\cdot\text{K}\left[2\times 150°\text{C/m}^2\times 1\ \text{m} - 12°\text{C}/1\ \text{m}\right] = -4608\ \text{W/m}^2$$ <

$$q_r(r_o) = A_r\, q''_r(r_{o},z) \qquad \text{where} \qquad A_r = 2\pi r_o(2z_o)$$

$$q_r(r_o) = -4\pi\times 1\ \text{m}\times 2.5\ \text{m}\times 4608\ \text{W/m}^2 = -144,765\ \text{W}$$ <

Note that the sign of the heat flux and heat rate in the positive r-direction is negative, and hence the heat flow is *into* the cylinder.

(d) The heat fluxes and the heat rates at end faces, $z = +z_o$ and $-z_o$, may be calculated using Fourier's law. The direction of the heat rate *in* or *out* of the end face is determined by the sign of the heat flux in the positive z-direction.

At the upper end face, $z = +z_o$: <

$$q''_z(r,+z_o) = -k\left.\frac{\partial T}{\partial z}\right)_{z_o} = -k\left[0+0+0+2dz_o\right]$$

$$q''_z(r,+z_o) = -16\ \text{W/m}\cdot\text{K}\times 2\left(-300°\text{C/m}^2\right)2.5\ \text{m} = +24,000\ \text{W/m}^2$$ <

$$q_z(+z_o) = A_z\, q''_z(r,+z_o) \qquad \text{where} \qquad A_z = \pi\left(r_o^2 - r_i^2\right)$$

$$q_z(+z_o) = \pi\left(1^2 - 0.2^2\right)\text{m}^2\times 24,000\ \text{W/m}^2 = +72,382\ \text{W}$$ <

Thus, heat flows out of the cylinder.

At the lower end face, $z = -z_o$: <

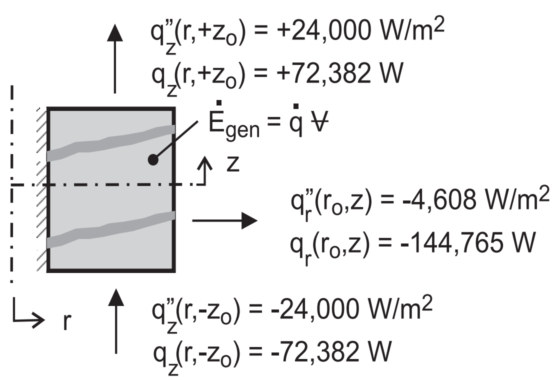
$$q''_z(r,-z_o) = -k\left.\frac{\partial T}{\partial z}\right)_{-z_o} = -k\left[0+0+0+2d(-z_o)\right]$$

$$q''_z(r,-z_o) = -16\ \text{W/m}^2\cdot\text{K}\times 2(-300°\text{C/m})(-2.5\ \text{m}) = -24,000\ \text{W/m}^2$$ <

$$q_z(-z_o) = -72,382\ \text{W}$$ <

Again, heat flows out of the cylinder.

(e) The heat rates from the surfaces and the volumetric heat generation can be related through an overall energy balance on the cylinder as shown in the sketch.



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PROBLEM 2-40 (Conti.)

$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$ where $\dot{E}_{gen} = \dot{q}\forall = 0$

$\dot{E}_{in} = -q_r\left(r_o\right) = -\left(-144,765\ W\right) = +144,765\ W$ <

$\dot{E}_{out} = +q_z\left(z_o\right) - q_z\left(-z_o\right) = \left[72,382 - \left(-72,382\right)\right] W = +144,764\ W$ <

The overall energy balance is satisfied.

COMMENTS: When using Fourier's law, the heat flux q''_z denotes the heat flux in the positive z-direction. At a boundary, the sign of the numerical value will determine whether heat is flowing into or out of the boundary.