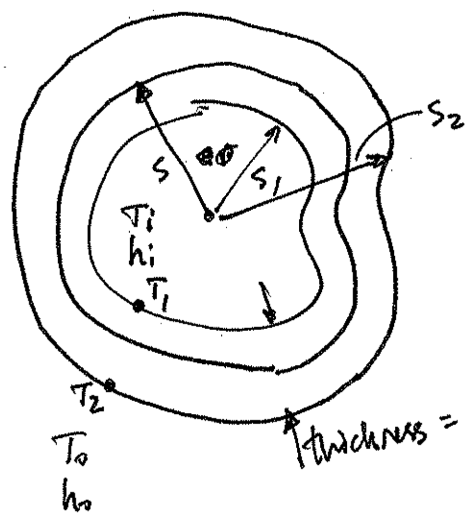
Generalized One-dim. H.T.

NOT COVERED IN THIS TEXT

Note $s = f(\theta)$

"but we don't need to specify how exactly"

Consider:



thickness $= s_2 - s_1$ = const. $\neq$ fn (θ)

Find Temp. Dist. in & Heat Transfer through the cylinder.

$$Q = \text{const.} = \underbrace{q_s}_{\text{H. flux at } s} \underbrace{A(s)}_{\text{Area at } s} \qquad q_s = -k\frac{dT}{ds}$$

$$-k\frac{dT}{ds}A(s) = \underset{Q}{\cancel{\text{const.}}}$$

What is this? Recall: $Q = \frac{\Delta T}{R_{th}}$

$$-k\frac{dT}{ds}A(s) = \frac{T_1 - T_2}{R_{th}}$$

$$-R_{th}\int_{T_1}^{T_2}\frac{dT}{T_1 - T_2} = \int_{s_1}^{s_2}\frac{ds}{k\,A(s)}$$

$$\boxed{R_{th} = \frac{1}{k}\int_{s_1}^{s_2}\frac{ds}{A(s)}} \quad \text{for const. } k$$

Generalized conductive thermal resistance

Total heat transfer is . . .

$$Q = \frac{T_i - T_o}{R_i + R + R_o}$$

$$R_i = \frac{1}{h_i A(s_1)}$$

$$R_o = \frac{1}{h_o A(s_2)}$$

$$R = \frac{1}{k} \int_{s_1}^{s_2} \frac{ds}{A(s)}$$

$\boxed{\text{Relate this to overall H.T. coeff.}}$

$$Q = U A (T_i - T_o)$$

$$\frac{1}{UA} = R_i + R + R_o$$

What area do you pick?
This is just a definition . . . so

let

$$UA = U_i A(s_1)$$

$$= U_o A(s_2)$$

U is dependent on area picked

$$\frac{1}{R_i + R + R_o} = U_o A(s_2)$$

$$\frac{1}{U_o} = \frac{A(s_2)/A(s_1)}{h_i} + \frac{A(s_2)}{k} \int_{s_1}^{s_2} \frac{ds}{A(s)} + \frac{1}{h_o}$$

~~Total heat flowing is const.~~

~~$Q = .$~~

To get temp. dist. . . . recall

$$Q = \frac{T_1 - T_2}{R_{th}} = \frac{T_1 - T_2}{\frac{1}{k} \int_{s_1}^{s_2} \frac{ds}{A(s)}}$$

$$-k \frac{dT}{ds} A(s) = Q = \text{const.}$$

$$\int_{T(s)}^{T_2} dT = \int_{s}^{s_2} -\frac{1}{k} Q \frac{ds}{A(s)}$$

or just T

$$T_2 - T = -\frac{Q}{k} \int_{s}^{s_2} \frac{ds}{A(s)}$$

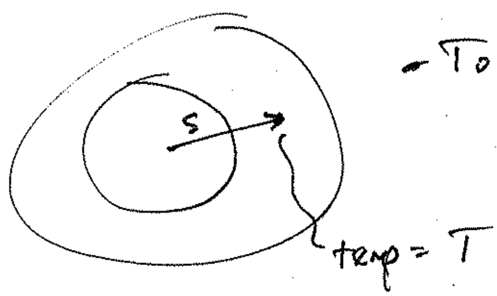
$$Q = \frac{T - T_2}{\frac{1}{k} \int_{s}^{s_2} \frac{ds}{A(s)}}$$

Other ways to write q ...

$$q = \frac{T_i - T_1}{\frac{1}{h_i A(s_1)}} \quad \text{or} \quad q = \frac{T_2 - T_o}{\frac{1}{h_o A(s_2)}}$$

or

Consider writing this from 's' to position outside of cylinder



$$q = \frac{T - T_o}{\frac{1}{k}\int_s^{s_2}\frac{ds}{A(s)} + \frac{1}{h_o A(s_2)}}$$

Get the same thing by ...

$$q\,\frac{1}{k}\int_s^{s_2}\frac{ds}{A(s)} = T - T_2$$

$$T_2 = T - \frac{q}{k}\int_s^{s_2}\frac{ds}{A(s)}$$

$$q = \frac{-Q * \frac{1}{k}\int \bigcirc + T - T_o}{\frac{1}{h_o A(s_2)}}$$

$$q\left[\frac{1}{k}\int_s^{s_2}\frac{ds}{A(s)} + \frac{1}{h_o A(s_2)}\right] = T - T_o$$

$$q = \frac{T - T_o}{\frac{1}{k}\int_s^{s_2}\frac{ds}{A(s)} + \frac{1}{h_o A(s_2)}}$$

This can be solved for T ... when a specific A is used...

E.G.: $Q = U_o A(s_2)(T_i - T_o)$

$$\frac{T - T_o}{\frac{1}{k}\int_{s_0}^{s_2}\frac{ds}{A(s)} + \frac{1}{h_o A(s_2)}} = \cancel{\frac{A(s_2)(T_i - T_o)}{\frac{A(s_2)/A(s_1)}{h_i} + \frac{A(s_2)}{k}\int_{s_1}^{s_2}\frac{ds}{A(s)} + \frac{1}{h_o}}}$$

$$\boxed{\frac{T - T_o}{T_i - T_o} = U_o\left[\frac{A(s_2)}{k}\int_{s_0}^{s_2}\frac{ds}{A(s)} + \frac{1}{h_o}\right]}$$

NOTE !!
This is an alternative way to get 1-Dim. temp. distributions

~~To see how this works~~

To see how this works ~~...~~ try it out in

spherical geometry

$$\frac{1}{U_o} = \frac{A(s_2)/A(s_1)}{h_i} + \frac{A(s_2)}{k}\int_{s_1}^{s_2}\frac{ds}{A(s)} + \frac{1}{h_o}$$

$$A(s) = 4\pi R^2$$

$$A(s_1) = 4\pi R_1^2$$

$$A(s_2) = 4\pi R_2^2$$

$$\frac{1}{U_o} = \frac{4\pi R_2^2 / 4\pi R_1^2}{h_i} + \frac{\cancel{4\pi} R_2^2}{k}\int_{R_1}^{R_2}\frac{ds}{\cancel{4\pi} R^2} + \frac{1}{h_o}$$

$$-\frac{1}{R}\Big|_{R_1}^{R_2} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{R_2}{R_2} = \left(\frac{R_2}{R_1} - 1\right)\frac{1}{R_2}$$

$$\boxed{\frac{1}{U_o} = \frac{R_2^2/R_1^2}{h_i} + \frac{R_2}{k}\left[\frac{R_2}{R_1} - 1\right] + \frac{1}{h_o}}$$

Now get Spherical temp distribution

$$\frac{T-T_0}{T_i-T_0} = U_0\left[\frac{4\pi R_2^2}{k}\underbrace{\int_R^{R_2}\frac{dR}{4\pi R^2}}_{\left(\frac{R_2}{R}-1\right)\frac{1}{R_2}} + \frac{1}{h_0}\right]$$

$$\boxed{\frac{T-T_0}{T_i-T_0} = U_0\left[\frac{R_2}{k}\left(\frac{R_2}{R}-1\right) + \frac{1}{h_0}\right]}$$

Where is k?

NOTE ... RESULTS FOR RECTANGULAR & CYLINDRICAL 1-DIM.

$$\left.\frac{1}{U_0}\right|_{rect.} = \frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_0} \qquad \left.\frac{T-T_0}{T_i-T_0}\right|_{rect.} = U_0\left[\frac{x_2-x}{L} + \frac{1}{h_0}\right]$$

$$\left.\frac{1}{U_0}\right|_{cyl.} = \frac{R_2/R_1}{h_i} + \frac{R_2}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_0} \qquad \left.\frac{T-T_0}{T_i-T_0}\right|_{cyl.} = U_0\left[\frac{R_2}{k}\ln\left(\frac{R_2}{R}\right) + \frac{1}{h_0}\right]$$