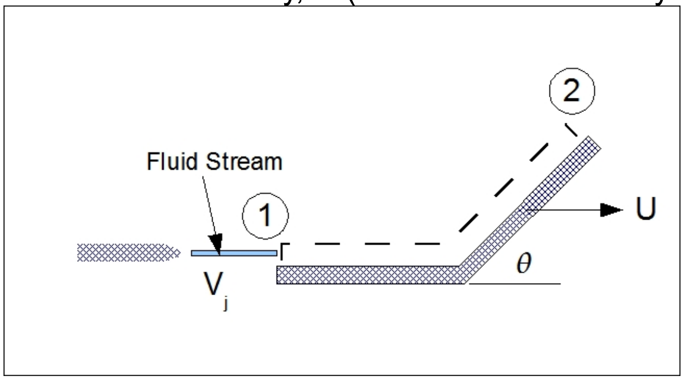
ENGR 3443 Fluid Mechanics

Force on a Moving Vane

Problem

Find the force exerted by the fluid jet (**V_j** relative to a stationary observer) on the vane moving with constant velocity, **U** (relative to a stationary observer). Note that V_j is greater than U.



Assume a fluid density of ρ and a jet area of A.

Solution

The momentum equation is

$$\sum \vec{F} = \frac{\partial}{\partial t} \int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \vec{V} \cdot d\vec{A}$$

Note that the ***V*** 's in the momentum equation must be relative to the CV. The first term on the right hand side will be zero as long as the momentum of the CV is not changing (since the acceleration of the CV is zero in this case, the momentum of the CV is not changing here).

Conservation of Mass is

$$\int_{inlets} \rho \vec{V} \cdot d\vec{A} = \int_{outlets} \rho \vec{V} \cdot d\vec{A} = \rho (V_j - U) A$$

Since ρ and A do not change from *1* to *2* the speed of the fluid leaving and entering the CV is the same ($V_j - U$). The momentum equation can be rewritten as

$$\sum \vec{F} = \dot{m} \left(\vec{V}_2 - \vec{V}_1 \right)$$

Equation 1

Now to find ***V_2*** and ***V_1***.

$$\vec{V}_1 = (V_j - U)\,\hat{i}$$

$$\vec{V}_2 = (V_j - U)\left[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$$

Inserting these expressions into Eq. (1) gives:

$$\Sigma\,\vec{F} = \rho\left(V_j - U\right)^2 A\left[(\cos\theta - 1)\,\hat{i} + \sin\theta\,\hat{j}\right]$$

Now identifying the components of the force

$$F_x = -\rho\left(V_j - U\right)^2 A\left[(1 - \cos\theta)\right]$$

$$F_y = \rho\left(V_j - U\right)^2 A\left[(\sin\theta)\right]$$

Note that the x-comp. will always be to the left and the y-comp. will always be up. To see the effects of changing the vane angle one may plot F_x and F_y.

Note that both forces vanish when θ is zero and are maximized when θ is $\pi/2$.