Exact Eqs:

$$dz = M(x,y)\,dx + N(x,y)\,dy$$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \text{exact differential}$$

$$\boxed{\text{Note: } \frac{\partial M}{\partial y} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial N}{\partial x} = \frac{\partial^2 f}{\partial y \partial x}}$$

If $dz = 0$, this is an exact equation which may be solved by:

$$\frac{\partial f}{\partial x} = M$$

$$\int \partial f = \int M\,\partial x$$

"constant" is a function of y

$$f(x,y) = \int M\,dx + g(y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\int M\,dx + g(y)\right]$$

N(x,y)

$$= \frac{\partial}{\partial y}\int M\,dx + g'(y)$$

$$g'(y) = N(x,y) - \frac{\partial}{\partial y}\left[\int M\,dx\right]$$

Integrate to find $g(y)$, then $f(x,y)$ will be Known.