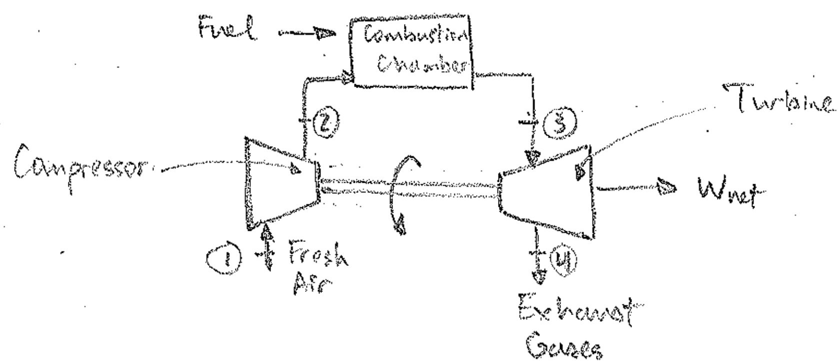
CH. EIGHT
GAS POWER CYCLES

8.8 THE BRAYTON CYCLE

8.A BASICS

Designed late 1870's, now used for Gas-Turbine Engines...



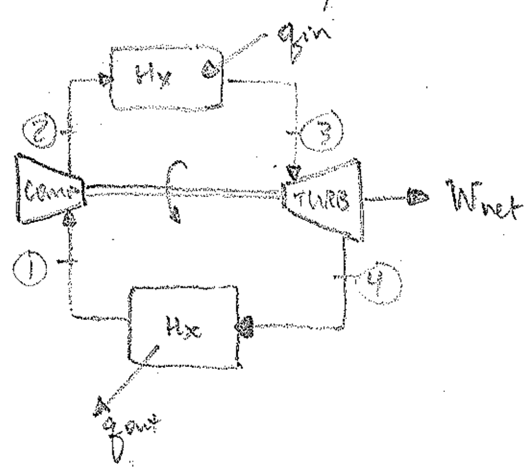
NOTE: OPEN CYCLE

1 → 2 Fresh air is compressed ⇒ T and p raised

2 → 3 High-pressure air has temp. raised significantly by Q from combustion of fuel

3 → 4 High-pressure, high-temp air expands through turbine producing work

BRAYTON CYCLE usually modelled as closed cycle



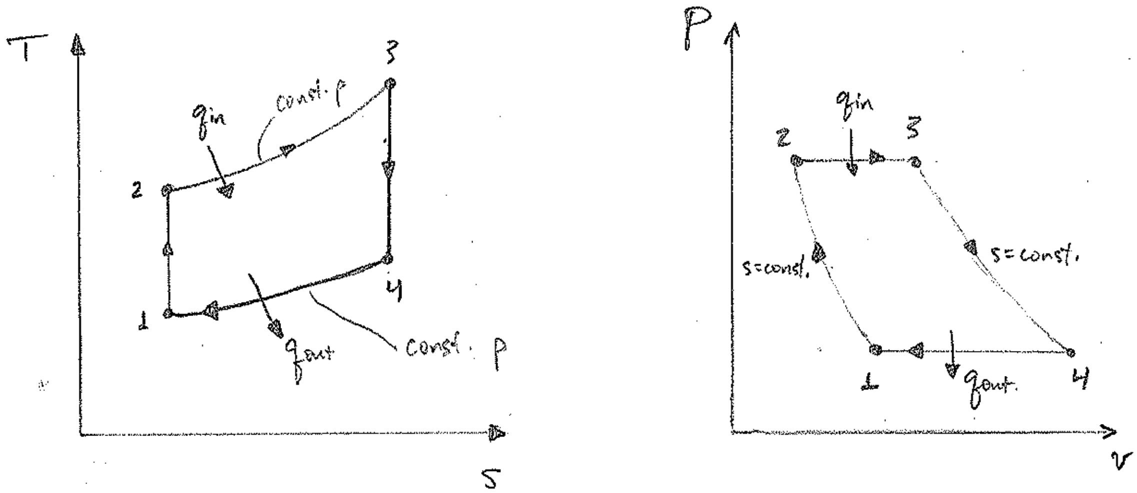
Idealized Brayton Cycle.

1 → 2 Isentropic compression (compressor)

2 → 3 constant p heat addition

3 → 4 Isentropic expansion (turbine)

4 → 1 const. p heat rejection



8.B Therodynamic Cycle Analysis

For SSSF processes and ignoring ΔKE and ΔPE

$$q - w = h_{exit} - h_{inlet}$$

① Compressor: $(1 \to 2)$

$$-w = h_2 - h_1$$

$-w =$ work input from turbine to compressor

$$\boxed{w_{comp} = h_2 - h_1}$$

② Combustion: $(2 \to 3)$

$$\boxed{q_{in} = h_3 - h_2 = C_p (T_3 - T_2)}$$

③ Turbine: $(3 \to 4)$

$$+w = h_3 - h_4$$

$$\boxed{w_{turb.} = h_3 - h_4}$$

$$\boxed{w_{net} = w_{turb} - w_{comp}}$$

Note: $\dfrac{w_{comp}}{w_{turb}} \equiv$ back work ratio

relatively large compared to steam power plants

④ Hx: $(4 \to 1)$

$$-q = h_4 - h_1 \Rightarrow \boxed{q_{out} = h_4 - h_1 = C_p (T_4 - T_1)}$$

Thermal efficiency of the cycle

$$\eta_{th.} \equiv \frac{w_{net}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$$\eta_{th} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{T_1}{T_2}\frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)}$$

Note:

$1 \to 2$ isentropic $\Rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$ $\qquad k \equiv \frac{C_p}{C_v}$

$3 \to 4$ isentropic $\Rightarrow \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k}$

and, $P_2 = P_3$, $P_1 = P_4$ $\Rightarrow$ thus

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

Define

$$r_p \equiv \frac{P_2}{P_1} = \frac{P_3}{P_4} = \text{pressure ratio}$$

Therefore

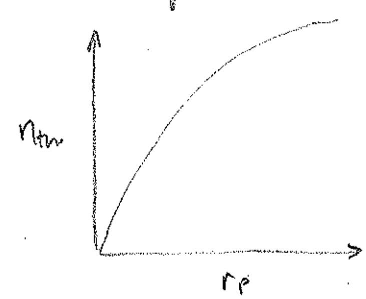
$$\frac{T_2}{T_1} = r_p^{(k-1)/k}$$

And

$$\eta_{th.} = 1 - \frac{1}{r_p^{(k-1)/k}} \frac{\left(\frac{T_4}{T_1} - 1\right)}{\left(\frac{T_3}{T_2} - 1\right)}$$

← ratio goes to 1

Fig 8-22
p. 398



Note

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Finally:

$$\boxed{\eta_{th.} = 1 - \frac{1}{r_p^{(k-1)/k}}}$$

Ideal case w/ const. C_p and C_v

$$5 \le r_p \le 20$$

$$\eta_{th} = 1 - f$$

where

$$f = \frac{1}{r_p^{(k-1)/k}}$$

note as p_2/p_1 increases $\Rightarrow$ f decreases $\Rightarrow$ η increases

The larger the difference between p_2 and p_1 the greater the cycle's thermal efficiency.

Example Text 8-5, p. 399

Power Plant, $r_p = 8$, $T_1 = 300K$, $T_3 = 1300K$

FIND: (a) gas temp. at exits of compressor and turbine

(b) back work ratio

(c) the thermal efficiency

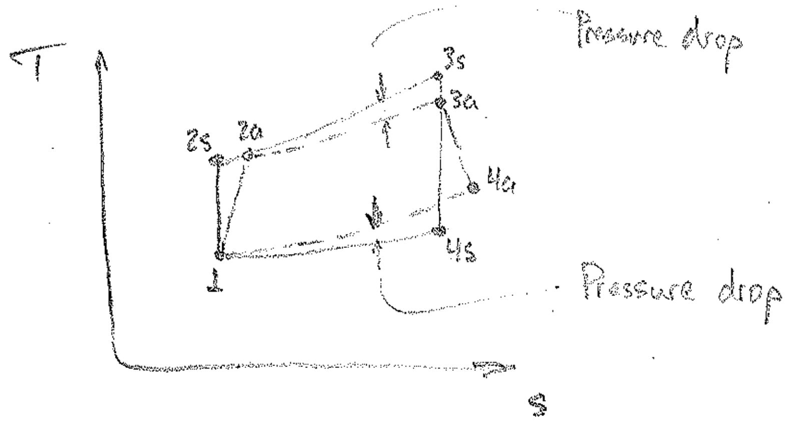
Self-Study Example

8.6 Actual Gas Turbine Cycles

Must account for irreversibilities of the turbine and compressor

$$\eta_c = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_{turb} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$



Tomorrow:

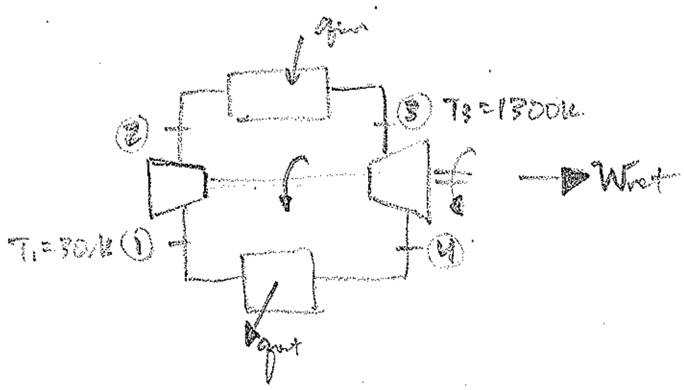
Example

8.9 Brayton Cycle w/ Regeneration

8.10 Brayton w/ Intercooling, Reheating, and Regeneration.

EXAMPLE Power plant, $r_p = 8$, $T_1 = 300$ K

$T_3 = 1300$ K



$\eta_{comp.} = 0.8$

$\eta_{turb.} = 0.85$

FIND: (a.) The air temps. T_2, T_4

(b.) The back work ratio

(c.) The thermal efficiency

SOLUTION:

(a) Consider ① → ② $\frac{P_2}{P_1} = 8$

Note $p_1 = 100$ kPa ⇒ atmospheric press.

For the isentropic case

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1}$$

Assume $k = 1.4$ then

$$\frac{T_2}{T_1} = (8)^{.4/1.4} =$$

Now,

$$T_{2s} = T_1 (8)^{.4/1.4} = 300\text{ K}\,(8)^{.4/1.4} = \underline{543}\text{ K}$$

<u>State 1</u>

~~$p_1 = 100$ kPa~~

$\underbrace{T_1 = 300\text{ K}}$

$h_1 = 300.19$ kJ/kg

<u>State 2</u>

~~$p_2 = 800$ kPa~~

$\underbrace{T_{2s} = 543}$

$h_{2s} = 547.5$ kJ/kg

} Ideal-gas air tables... (A-17)

Find compressor work ...

$$W^{\text{ideal}}_{\text{comp.}} = h_{2s} - h_1 = 547.5 - 300.19\ \frac{\text{kJ}}{\text{kg}}$$

$$= 247.3\ \text{kJ/kg}$$

Now,

$$\eta_{\text{comp.}} = 0.80 = \frac{W^{\text{ideal}}_{\text{comp.}}}{W^{\text{act.}}_{\text{comp.}}}$$

$$W^{\text{actual}} = W^{\text{ideal}}_{\text{comp}} / 0.8 = h_{2a} - h_1$$

$$= 309.1 = h_{2a} - h_1$$

Thus,

$$h_{2a} = 309.1 + 300.19 = 609.3\ \text{kJ/kg}$$

$$\boxed{T_{2a} = 602.2\text{ K}}$$

Same process for turbine

Table A-2 at AVE temp. of 1000 K

$$T_{4s} = T_3/(8)^{k-1/k} = 1300/(8)^{1.34-1/1.34} = 771\ K$$

<u>State 3</u>

$T_3 = 1300K$

$h_3 = 1395.97 \frac{KJ}{kg}$

<u>State 4s</u>

$T_{4s} = 771\ K$

$h_{4s} = 790.2 \frac{kJ}{kg}$ linear interpolation

$$W_{turb.}^{ideal} = h_3 - h_{4s} = 1395.97 - 790.2 = 605.8 \frac{KJ}{kg}$$

Now,

$$\eta_{turb.} = \frac{w^{act}}{w^{ideal}} \Rightarrow w^{act.} = 0.85(605.8) = 514.9\ KJ/kg$$

$$W_{turb.}^{act.} = 514.9 = 1395.97 - h_{4a}$$

$h_{4a} = 881.1\ kJ/kg$ } Table A-17 interpolation

$$\boxed{T_{4a} = 853.5\ K}$$

(b.) Back work ratio : $r_{bw} = \frac{W_{comp}}{W_{turb.}}$

$$r_{bw} = \frac{309.1}{514.9} = \boxed{60\ \%}$$

60% of turbine output goes "back" to <u>compressor</u>

(c.) Thermal efficiency

<u>Ideal</u>: $\eta_{th} = 1 - \frac{1}{8^{(1.4-1)/1.4}} = .448$

<u>Actual</u>: $\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{W_{turb} - W_{comp}}{h_3 - h_{2a}} = \frac{514.9 - 309.1}{1395.97 - 609.8}$

$$\boxed{= 26.2\ \%}$$

8.C BRAYTON CYCLE WITH REGENERATION

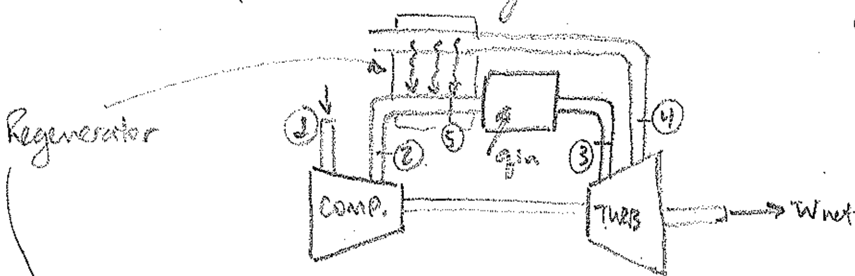
In last example $T_4 = 854K >> T_2 = 602K$

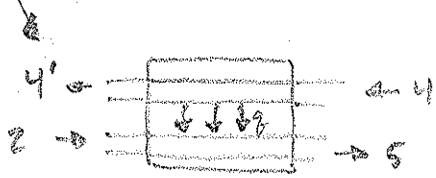
This means heat may be transferred from pt. 4 to pt. 2. This decreases the heat rejection (q_{out}).

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$$

Hence efficiency of cycle is increased.

Heat Exchange via a "regenerator"





1st Law

$$q_{regen} = h_4 - h_{4'}$$

$$q_{regen} = h_5 - h_2 \quad \text{} \} \text{ actual heat transfer}$$

If $h_{4'} \equiv h_2$, then the Hx is perfect (ideal)

$$q_{regen} = h_4 - h_2 \quad \} \text{ ideal heat trans.}$$

Thus, an "effectiveness" ε can be defined as

$$\varepsilon \equiv \frac{h_5 - h_2}{h_4 - h_2}$$

$$\varepsilon \cong \frac{T_5 - T_2}{T_4 - T_2} \quad \} \text{ const. } C_p \text{ assumption}$$

When c_p is constant ...

$$\eta_{th} = 1 - \left(\frac{T_1}{T_3}\right) r_p^{(k-1)/k}$$

<u>Effect of a Regenerator</u>:

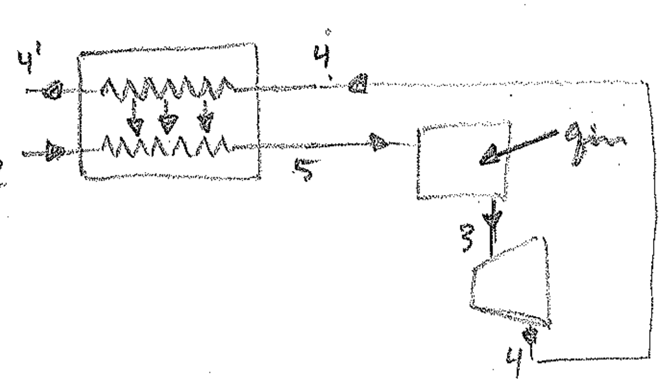
Decreases the amount of heat needed in the combustion chamber, by the amount

$$\boxed{q_{regen,act.} = h_5 - h_2}$$

<u>Example</u> (Same as previous example except with an $\varepsilon = 80\%$ regenerator) Note: the cycle thermal efficiency is increased from 26.2 % → 36.9 % !!

Note, for a perfect regenerator ($\varepsilon = 1.00$)

start here!!



$q_{out} = q_{in}$

$h_4 - h_{4'} = h_5 - h_2$

$h_5 = h_4 - h_{4'} + h_2$

for complete heat extraction $h_{4'} = h_2$

Hence,

$$\underline{\underline{h_5 = h_4}}$$

$$q_{in} = h_3 - h_5 = h_3 - h_4$$

$$W^{ideal}_{turb.} = h_3 - h_4$$

$q_{in} = W^{ideal}_{turb.}$

hmm...

8.D BRAYTON WITH INTERCOOLING, REHEATING, AND REGENERATION

Compressor work ≡ W_{comp} may be reduced by compressing in stages with an intercooler between stages. (Isothermal process)

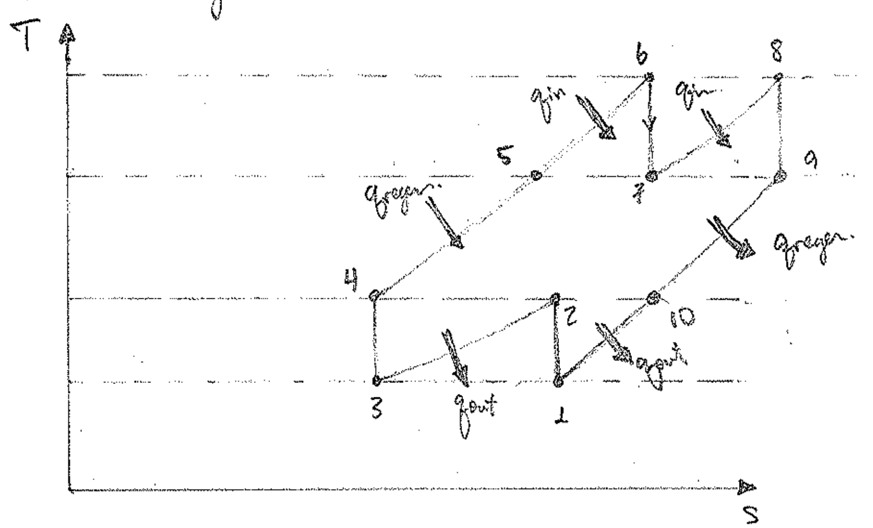
Turbine work ≡ $W_{turb.}$ may be increased by expanding in stages with a reheater between stages (Isothermal process)

Intercooler ⇒ increases T_2
Reheater ⇒ increases T_4
} use regenerator to take advantage of temp. difference $(T_4 - T_2)$

See Fig. 8-43 p. 405

↳ These power systems can get complicated pretty quickly

Typical T-s diagram



Multistage Compressors & Turbines

$w_{comp.}$ is minimum when $\frac{P_2}{P_1} = \frac{P_4}{P_3}$ $w_{comp.}^{MIN}$

$w_{turb.}$ is maximum when $\frac{P_6}{P_7} = \frac{P_8}{P_9}$ $w_{turb.}^{max}$

This means

$$r_{bw} = \frac{w_{comp.}^{MIN.}}{w_{turb.}^{MAX.}} \Rightarrow r_{bw}^{MIN.}$$

⇒ But, using ~~reheaters~~ ~~and~~ an intercoolers decreases temp. at which heat is added

⇒ using reheater increases temp. at which heat is rejected . . .

These effects combined decrease thermal efficiency – thus must use a "regenerator".

EXAMPLE Overall pressure ratio of 8

$$\frac{P_4}{P_1} = 8 \ ; \qquad \frac{P_2}{P_1} = \frac{P_4}{P_3} \Rightarrow \frac{P_2 P_3}{P_4 P_1} = 1$$

$$\frac{P_2 P_3}{\cancel{P_4} P_1} \frac{\cancel{P_4}}{P_1} = 8 \qquad \text{Note ;} \qquad P_2 = P_3$$

$$\left(\frac{P_2}{P_1}\right)^2 = 8 \Rightarrow \boxed{\frac{P_2}{P_1} = \frac{P_4}{P_3} = \sqrt{8}}$$

Part (a)

For compressor stages

$$T_{2s} = T_1 \left(\sqrt{8}\right)^{.4/1.4} = 300\left(\sqrt{8}\right)^{.4/1.4} = 403.8 \text{ K}$$

$$T_{4s} = T_{2s} = 403.8 \text{ K}$$

States 1 & 3	States 2s & 4s	
$T_1 = T_3 = 300$ K	$T_{2s} = T_{4s} = 403.8$	
$h_1 = h_3 = 300.19 \frac{kJ}{kg}$	$h_{2s} = h_{4s} = 404.8$ kJ/kg	linear interp. A-17 ...

Thus,

$$w_c^{ideal} = (h_{2s} - h_1) + (h_{4s} - h_3)$$

$$= 2(h_{2s} - h_1) = 2(404.8 - 300.19) =$$

$$= 209.2 \text{ kJ/kg}$$

Note,

$$\eta_{comp} = 0.8 = \frac{w_c^{ideal}}{w_c^{act.}} \Rightarrow w_c^{act.} = \frac{209.2/2}{.8} = 130.75 \frac{kJ}{kg}$$

to 'each' compressor

$$130.75 = h_{2a} - h_1 \Rightarrow h_{2a} = 130.75 + 300.19 \Rightarrow$$

$$h_{2a} = 430.9 \text{ kJ/kg}$$

$$\boxed{T_{2a} = T_{4a} = 429.5 \text{ K}}$$

For turbine stages (Turbine inlet 1300 K)

$$T_{7s} = \frac{T_6}{(\sqrt{8})^{\frac{1.34-1}{1.34}}} = \frac{1300}{(\sqrt{8})^{.34/1.34}} = 998.6\ K$$

States 6 & 8 — States 7s & 9s

$T_6 = T_8 = 1300K$ — $T_{7s} = T_{9s} = 998.6\ K$

$h_6 = h_8 = 1395.97$ — $h_{7s} = h_{9s} = 1044.4$

each turb.

$$w_{turb}^{ideal} = h_6 - h_{7s} = 1395.97 - 1044.4 = 351.6\ \frac{kJ}{kg}$$

$$w_{turb}^{act} = (0.85)(351.6) = 298.9\ \frac{kJ}{kg}$$ each turbine

$$= h_6 - h_{7a} \Rightarrow h_{7a} = h_6 - w_t^{act}$$

$$= 1395.97 - 298.9$$

$$= 1097.1\ kJ/kg$$

$$\boxed{T_{7a} = T_{9a} = 1045\ K}$$

(b) Find ~~compressor~~ back-work ratio

$$r_{bw} = \frac{w_{comp}}{w_{turb.}} = \frac{\cancel{2}(130.75)}{\cancel{2}(298.9)} = \boxed{43.7\ \%}$$

(c) Find thermal efficiency

$$\eta_{th.} = \frac{w_{net}}{q_{in.}}$$

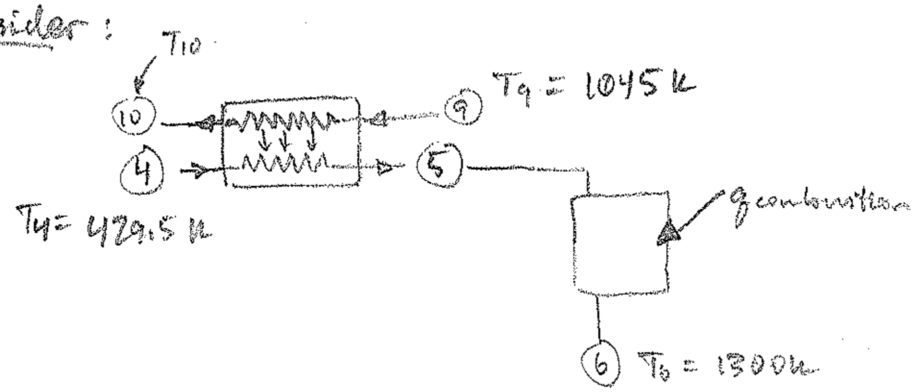
$$w_{net.} = w_{turb} - w_{comp.} = 2(298.9 - 130.75)$$

$$= 336.3\ kJ/kg$$

What about q_{in} ?

$$q_{in} = q_{combustion} + q_{reheat} \quad \} \text{Total heat input}$$

Consider:



For a perfect ($\varepsilon = 1.00$) regenerator $\quad T_{10} = T_4 = 429.5$ K

$\underbrace{h_{10} = h_4}$ $= 430.9$ $\frac{kJ}{kg}$

$$\varepsilon = \frac{h_5 - h_4}{h_9 - h_{10}} = 1$$

$$h_9 - h_{10} = h_5 - h_4$$

$$h_5 = h_9 - \cancel{h_{10}} + \cancel{h_4}$$ Recall $h_9 = 1097.1$

$$h_5 = 1097.1 \text{ kJ/kg}$$

Now,

$$q_{combust.} = h_6 - h_5 = 1395.97 - 1097.1$$
$$= 298.9 \text{ kJ/kg}$$ hmmm ....

What about q_{reheat}?

$$q_{reheat} = h_8 - h_7 = h_6 - h_7$$
$$= 1395.97 - 1097.1 = 298.9 \text{ kJ/kg}$$

hmm ....

Thus,

$$q_{in} = (298.9)\,2 = 598 \text{ kJ/kg}$$

And $\eta_{th.} = \frac{w_{net}}{q_{in}} = \frac{336.3}{598} = \boxed{56.3\ \%}$

8.9 (8.11 in text) IDEAL JET PROPULSION CYCLES

Basically just Brayton cycle. Turbine produces just enough work to run compressor.

Explain via figure 8-48 p.409

1→2 Diffuser - air slowed down, pressure goes up a little

2→3 Compressor - temp. and pressure go up significantly

3→4 Burner - jet fuel burned - heats up air at high pressure

4-5 Turbine - Air is expanded - only enough to run the compressor

5→6 Nozzle - High pressure air is accelerated through a nozzle to atmospheric pressure.

Idea here is to have the air speed much higher at the exit than the inlet

$$\text{Thrust Force} = F = \dot{m}\,(V_{exit} - V_{inlet})$$

mass flow rate of air

$$\text{Propulsive Power} = \dot{W}_P = \dot{m}\,(V_{exit} - V_{inlet})\,V_{AIRCRAFT}$$

Propulsive efficiency defined as

$$\eta_P \equiv \frac{\text{propulsive power}}{\text{energy input rate}} = \frac{\dot{W}_P}{\dot{Q}_{in}}$$

EXAMPLE

$V_{aircraft} = 850$ ft/s

$P_1 = 5$ psia $\quad T_4 = 2000\,°F$

$T_1 = -40\,°F$

$\dot{m} = 100$ lbm/s

$r_p = 10$

$C_p = 0.240 \ \frac{Btu}{lbm \cdot °R}$

FIND: (a) T_5, P_5 (at turbine exit)

(b) V_6 (at nozzle exit)

(c) η_P

SOLUTION:

(a) 1-2: Diffuser

assume ≈ 0

$$\cancel{q}^{0} - \cancel{w}^{0} = h_2 - h_1 + \cancel{\frac{V_2^2 - V_1^2}{2}}$$

Assume: $h_2 - h_1 = C_p (T_2 - T_1)$.

Thus: $T_2 = \frac{V_1^2}{2 C_p} + T_1 = \frac{(850\ ft/s)^2}{2\,(.240\ \frac{Btu}{lbm \cdot °R})} \left(\frac{1\ Btu/lbm}{25037\ ft^2/s^2} \right) + 420\,°R$

$T_2 = 480.1\,°R$ Still about 20°F

For isentropic case:

assume = 1.4

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^{k/1-k} \Rightarrow P_2 = 8 \text{ psia}$$

2-3 : compresser ($\eta_c = 0.8$)

$$\frac{P_3}{P_2} = r_p = 10$$

$$P_3 = 10\,(8\,\text{psia}) = 80\text{ psia}$$

1.4 again!

$$T_{3s} = T_2 \left(\frac{P_3}{P_2}\right)^{k-1/k} = 926.9^\circ R$$

$$W_{comp}^{Ideal} = h_{3s} - h_2 = C_p(T_{3s} - T_2)$$

$$= \left(.240 \frac{Btu}{lbm\,^\circ R}\right)(926.9 - 480.1)^\circ R$$

$$= 107.2\ Btu/lbm$$

$$\eta_c = .8 = \frac{W_{comp}^{ideal}}{W_{comp}^{act.}} \Rightarrow W_c^{act} = \frac{107.2}{.8} = 134.\ \frac{Btu}{lbm}$$

$$134 = h_{3a} - h_2 = (0.24)(T_{3a} - T_2)$$

$$T_{3a} = \frac{134}{.24} + T_2 = \frac{134}{.24} + 480.1 = \underline{1038.6^\circ R}$$

$T_4 = 2460\ ^\circ R$ Given ...

$p_4 = 80$ psia const. p. heat addition

Recall;

$$W_{turb.} = W_{comp.}$$

$$h_4 - h_5 = h_{3a} - h_2$$

$$\cancel{C_p}(T_4 - T_5) = \cancel{C_p}(T_{3a} - T_2) \quad \Big| \quad T_5 = T_4 - T_{3a} + T_2$$

$T_5 = 2460°R - 1038.6°R + 480.1°R$

$= 1901.5\ °R$

still use 1.4 ?

Now,

$$\frac{P_5}{P_4} = \left(\frac{T_5}{T_4}\right)^{k/k-1} \Rightarrow P_5 = (80)\left(\frac{1901.5}{2460}\right)^{1.4/.4} = 32.5\ \text{psia}$$

(b) Consider the nozzle

$$\cancel{q}^{0} - \cancel{w}^{0} = h_6 - h_5 + \frac{V_6^2 - \cancel{V_5^2}}{2}$$ assume 0

$$V_6 = \sqrt{2c_p(T_5 - T_6)}$$

What is T_6 ;

atmospheric press = 5 psia

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = (1901.5)\left(\frac{5}{32.5}\right)^{.4/1.4}$$

$T_6 = 1113.9\ °R$

Thus,

$$V_6 = \sqrt{2(.24)(1901.5 - 1113.9)\left(\frac{25{,}039\ ft^2/s^2}{1\ Btu/lbm}\right)}$$

$$\boxed{= 3076.6\ ft/s}$$

(C.) Find η_P !

$$\dot{W}_P = \dot{m}\left(V_{exit} - V_{inlet}\right) V_{air}$$

$$= (60\ \text{lbm/s})(3076.6 - 850)\,\text{ft/s}\ \left(\frac{1\ \text{Btu/lbm}}{25{,}037\ \text{ft}^2/\text{s}^2}\right)$$

850 ft/s

$$= 7559\ \text{Btu/s}$$

$$\dot{Q}_{in} = \dot{m}(h_4 - h_3)$$

$$= \dot{m}\,C_p\,(T_4 - T_3)$$

$$= 100\ \frac{\text{lbm}}{\text{s}}\left(.24\ \frac{\text{Btu}}{\text{lbm}\,°\text{R}}\right)(2460 - 1038.6)\,°\text{R}$$

$$= 34{,}114\ \frac{\text{Btu}}{\text{s}}$$

Thus,

$$\eta_P = \frac{\dot{W}_P}{\dot{Q}_{in}} = \frac{7559}{34{,}114} = 22.2\%$$

Note, the remaining 77.8 % of energy was released as heat or KE that could not be used.