ENGR 3203

Notes on Calculating Work for Ideal Gases and Polytropic Processes

- Work done by system on boundary is:
  - $$W=\int_{V_1}^{V_2} p \; dV$$
  - This form is used for expansion and contraction of gases
  - Ideal Gases
    - **Ideal (Perfect) Gas Law**
      - $pV=n\bar{R}T$
      - $\bar{R}=8.314\,\frac{kJ}{kgmol-K}$
      - The gas constant for a particular gas (air in this example) may be found as
      - $$R_{air}=\frac{\bar{R}}{M_{air}}=\frac{8.314\,\frac{kJ}{kgmol-K}}{28.97\,\frac{kg}{kgmol}}=0.287\,\frac{kJ}{kg-K}$$
      - Rewrite the Ideal Gas Law in terms of mass and the gas constant for air in this example as
      - $p\;V=m_{air}R_{air}T$
    - **Isotropic (Constant Temp) Process**
      - For a constant temperature process in a closed system (i.e. mass is constant) – $pV = mRT=C$. Where C is a constant. Note C can be written as p_1V_1 or as p_2V_2.
      - $$W=\int_{V_1}^{V_2}\frac{C}{V}dV=C\ln\left(\frac{V_2}{V_1}\right)=p_1V_1\ln\left(\frac{V_2}{V_1}\right)$$
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    - **Polytropic process -** $pV^n=C$ **where *C* is a constant.**
      - these occur in ideal gases for various processes and the value of n changes depending on the type of process (e.g. n = 1 is a isotropic process).
      - Note that $p_1V_1^n=C\rightarrow p_1V_1=\frac{CV_1}{V_1^n}=CV_1^{1-n}$ This also holds for p_2V_2.
      - $$W=\int_{V_1}^{V_2}\frac{C}{V^n}dV=\frac{C}{1-n}(V_2^{1-n}-V_1^{1-n})=\frac{p_2V_2-p_1V_1}{1-n}$$