Statically Indeterminate Torsion

A shaft shown in Fig. 1 is subjected to a torsional load, ***T***. The ends of the shaft at *A* and *C* are not allowed to rotate – they are fixed.

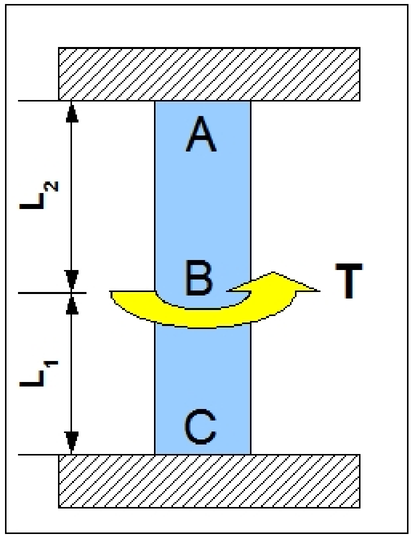

Figure 1

Fig. 2 shows a free body diagram (FBD) of the shaft with two support reactions shown, T_A and T_C. This figure also includes a z-axis. Summing moments about the z-axis (following the right hand rule) gives:

$$\Sigma M_z = 0 = T - T_A - T_C \Rightarrow T = T_A + T_C$$

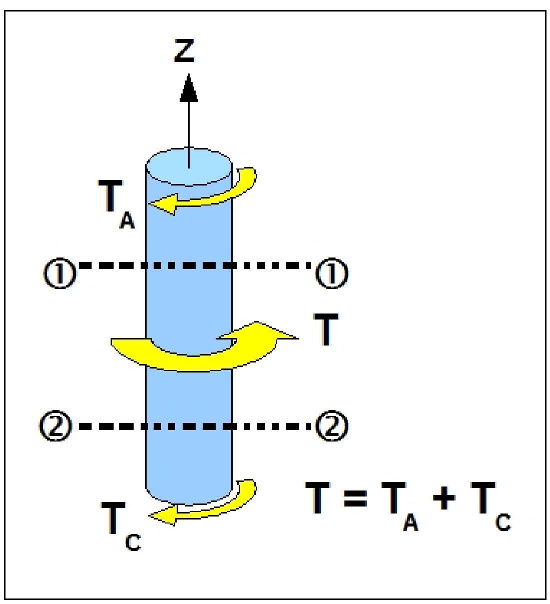

Figure 2

The determine the torsional load in the portion of the shaft between *A* and *B* consider section ①-① (shown in Fig. 2). To determine the equilibrium of the section from A to ①-① consider Fig. 3, which shows a FBD. Summing moments about the z axis:

$$\Sigma M_z = 0 = T_{AB} - T_A \Rightarrow T_{AB} = T_A$$

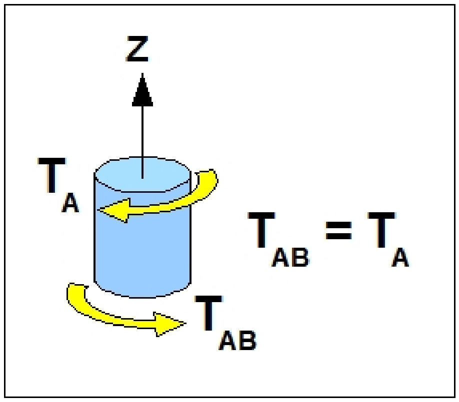

Figure 3

The determine the torsional load in the portion of the shaft between *B* and *C* consider section ②-② (shown in Fig. 2). To determine the equilibrium of the section from A to ②-② consider Fig. 4, which shows a FBD. Summing moments about the z axis:

$$\Sigma M_z = 0 = T_{BC} + T - T_A \Rightarrow T_{BC} = T_A - T$$

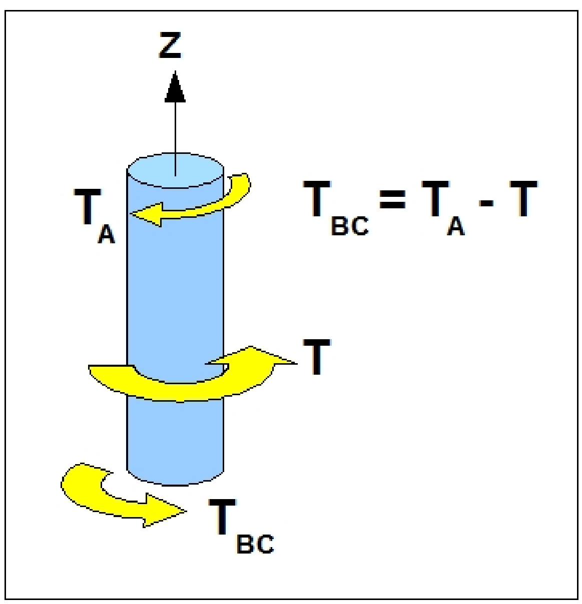

Figure 4

To find the support reactions T_A and T_C consider the angle of twist beginning from end *A*. The angle of twist at point *B* relative to point *A* is

$$\phi_{B/A} = \frac{T_{AB} L_2}{J_{AB} G_{AB}}$$

The angle of twist at point *B* relative to point *A* is

$$\phi_{C/B} = \frac{T_{BC} L_1}{J_{BC} G_{BC}}$$

The angle of twist between point A relative to C is

$$\phi_{C/A} = \phi_{B/A} + \phi_{C/B}$$

Finally, the shaft is constrained to not rotate at the ends, so,

$$\phi_{C/A} = 0 = \phi_{B/A} + \phi_{C/B}$$

By inserting the equations for $\phi_{B/A}$ and $\phi_{C/B}$ you can find T_A and T_C.