ENGR 2143 Strength of Materials
Shearing Stresses in a Glued Joint of a T-section Beam

Our goal is to calculate the sharing stress in joint *A* in the beam cross section shown in Fig. 1. Here the height of the joint, h_1, and the width of the joined member, b_1, are variable. The overall beam height is fixed at h_2, and the center section has a constant width b_2.

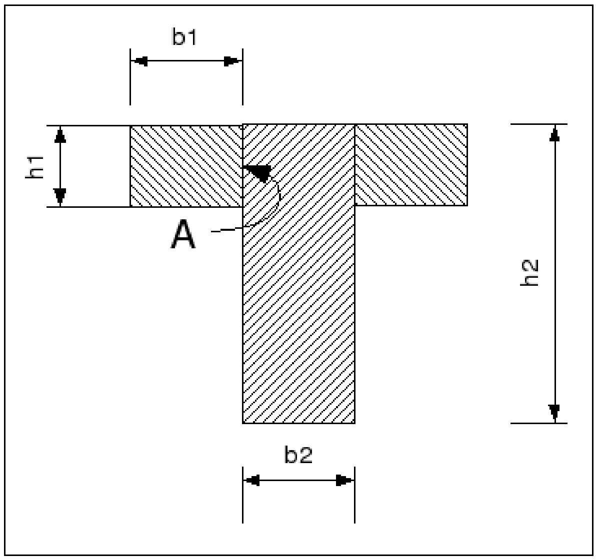

Figure 1: T cross section for calculation of shearing stress in joint A.

The shearing stress may be calculated as

$$\tau = \frac{VQ}{It}$$

where

V	=	shear load acting on the beam cross-section,
Q	=	first moment of an area A',
I	=	Area moment of inertia of cross-section,
t	=	thickness of the joint separating A' and remainder of the area.

To determine these we need the neutral axis (NA), location of the centroid of the cross-section. Before doing this please look at Fig. 2 which shows the correct choice for A'.

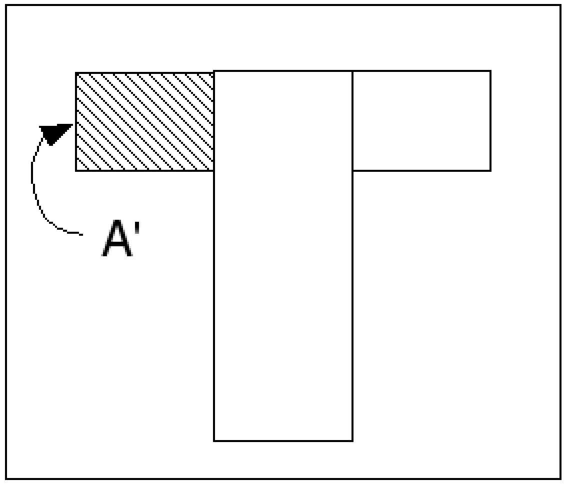

Figure 2: A' for the problem under consideration.

Through some amount of algebra the neutral axis may be found in terms of the parameters listed in Fig. 1 to be

$$\bar{y}=\frac{\left(1-\frac{h^{'}}{2}\right)b^{'}h^{'}+\frac{1}{4}}{b^{'}h^{'}+\frac{1}{2}}$$

Note that the value of $\bar{y}$ is measured from the bottom of the cross-section. Also note that the parameters h' and b' are defined by

$$h^{'}=\frac{h_1}{h_2}$$

$$b^{'}=\frac{b_1}{b_2}$$

So that when h' = 1 the section A' has the same height as the entire beam or when b' = 1 each of the two “T” sections have the same width as the center section.

The next step is to find the value of I about the NA. Again through a “little” algebra

$$I=\frac{b_2 h_2^{3}}{4}f(h^{'})$$

where f(h') is

$$f(h^{'})=\left[\frac{1}{12}+(\frac{4b^{'}}{3}+\frac{1}{4})h^{'}+(\frac{b^{'2}}{3}+\frac{1}{4}-2b^{'})h^{'2}+(\frac{13b^{'}}{12}+\frac{1}{16})h^{'3}+\frac{b^{'2}}{3}h^{'4}+\frac{b^{'3}}{3}h^{'5}\right]\frac{1}{(\frac{1}{2}+b^{'}h^{'})^{2}}$$

It is also possible to find Q as

$$Q=\overline{y}^{'}A^{'}=\frac{b_1 h_1 h_2}{4}\left[\frac{1-\frac{h^{'}}{2}}{\frac{1}{2}+b^{'}h^{'}}\right]$$

So finally the shearing stress is

$$\tau=\frac{V}{{h_2}^2}g(h^{'})$$

where

$$g(h^{'})=\frac{1}{f(h^{'})}\frac{1-\frac{h^{'}}{2}}{\frac{1}{2}+b^{'}h^{'}}$$

A graphical representation of the result for non-dimensionalized shear stress is shown in Fig. 3.

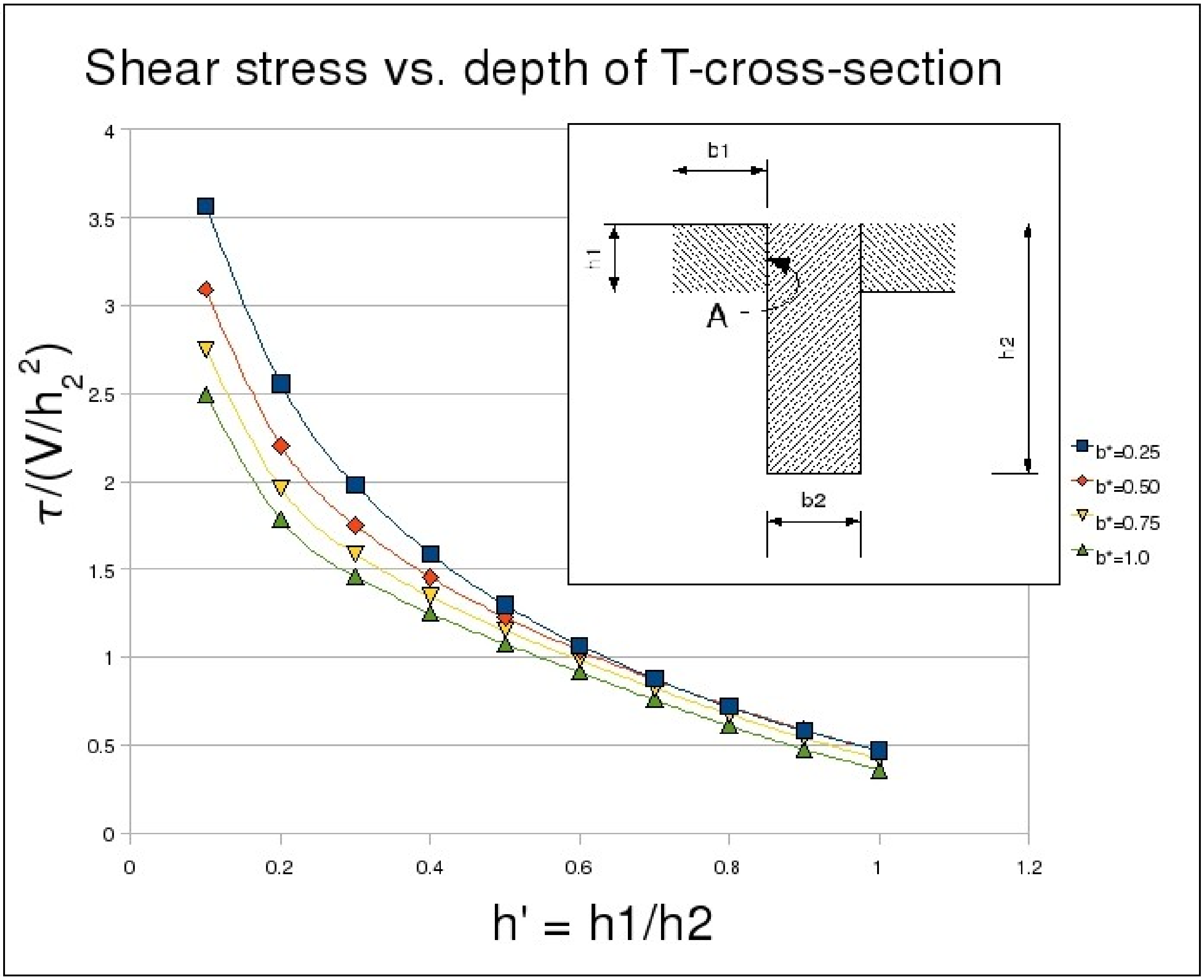

Figure 3: Non-dimensionalized shear stress for joint A for T cross-section shown.