Logarithms and Log Plots

The logarithm is defined as

$$\log_a x = b$$
$$a^b = x$$

Equation 1

The most common uses of the logarithms are with bases of *10* or *e*. Log base *10* is usually called a *common logarithm* and log base *e* is called a *natural logarithm*. It is important to know several laws of logarithms, such as

$$\log_a(xy) = \log_a(x) + \log_a(y)$$
$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$
$$\log_a(x^y) = y\log_a(x)$$
$$A^a = e^{a\,lnA}$$

Equation 2

The *e* that appears in Eq.(2) above is just a number and can be found as:

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots = 2.718281828459045$$

Equation 3

It is also convenient to know how to convert from one base to another. If you want to find $log_a x=b$ where *a* is any value, then from Eqs.(1)

$$a^b = x$$
$$\ln(a^b) = \ln(x) \rightarrow b\ln(a) = \ln(x) \rightarrow b = \frac{\ln(x)}{\ln(a)}$$

Equation 4

So *b* may be found just by taking the the ratio of the *log* of *x* to the *log* of the base where it should be noted that the ratio may use any base logarithms desired.

Logarithmic plots are commonly used to show data or other information where on a particular axis the scale needs to reflect several orders of magnitude. These plots may also be used in efforts to *linearize* data – to “flatten” it out so that a simple linear fit may be accomplished. The appearance of log plots may be achieved by simply taking the log of the data for the axis where a log scale is desired. Most of the time paper with log scales is available or software with the capability to use log scales is available. As an example consider a set of data

Table 1

x	***y***
1.1	5100
15.2	950
120	99.2
800	5.3
1500	0.11

This data is simply plotted in Figure One.

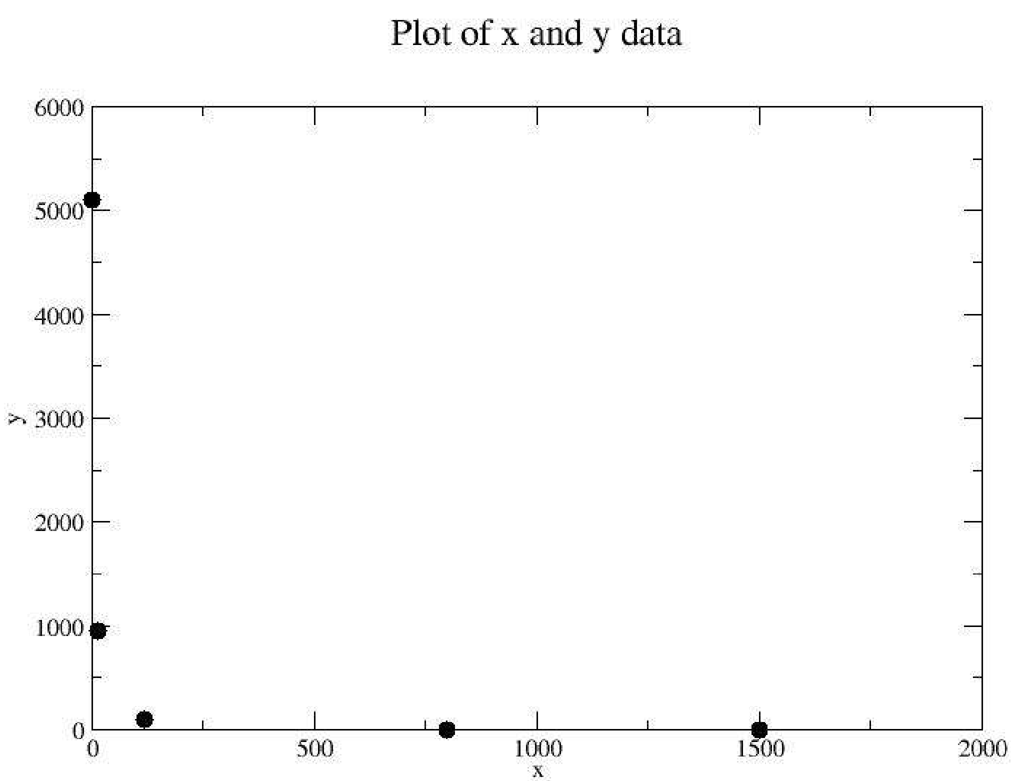

Figure 1 Plot of data from Table One.

This data is ripe for plotting in log-log format. So as an exercise let's try it out just by taking the logarithms of each x and y value ... this produces the following series of data:

Table 2

log(x)	***log(y)***
4.14E-002	3.71E+000
1.18E+000	2.98E+000
2.08E+000	2.00E+000
2.90E+000	7.24E-001
3.18E+000	-9.59E-001

Which is plotted in Figure Two

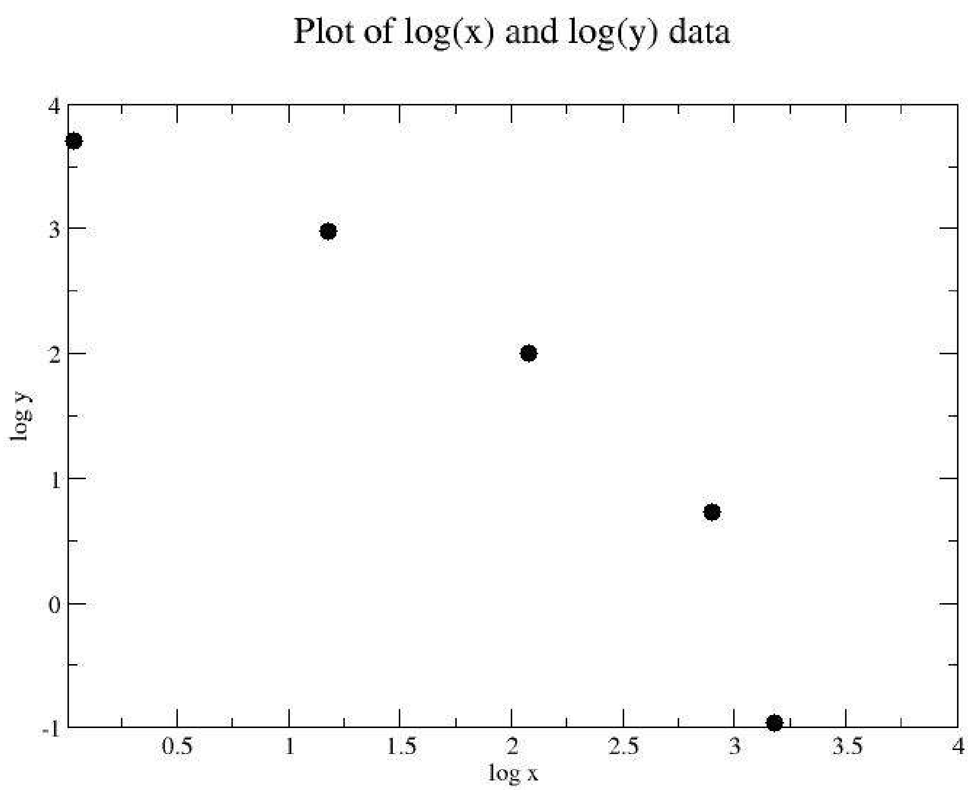

Figure 2 Data from Table Two.

Now finally showing the plots on a log-log basis is the best way. This is done in Figure Three.

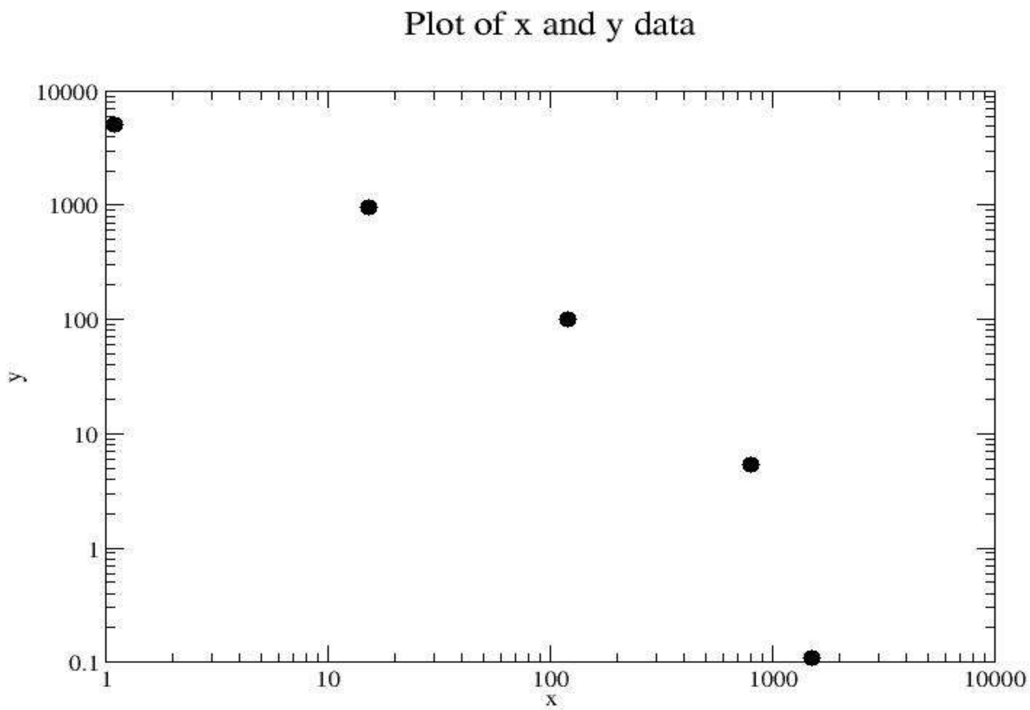

Figure 3 Data from Table One on a log-log basis.

In Figure 4 the data is plotted with a non-linear curve-fit.

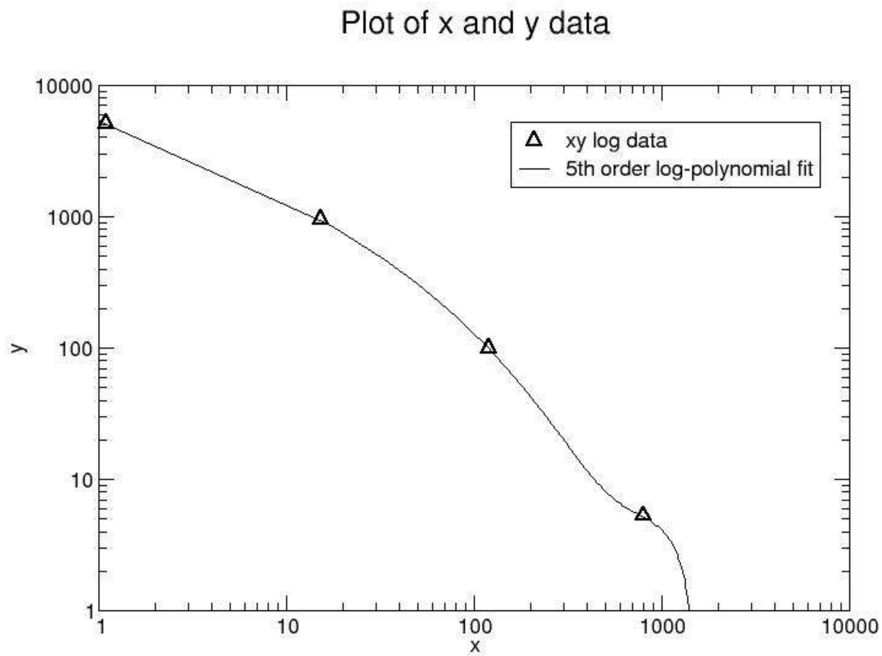

Figure 4 The data from Table One plotted with the curve-fit:

$$y = A_0 + A_1 \ln(x) + A_2(\ln(x))^2 + A_3(\ln(x))^3 + A_4(\ln(x))^4 + A_5(\ln(x))^5 \quad .$$