Engineering Physics Fundamentals

Introduction

This document is meant to help you attain (or review) some basic skills that physicists and engineers should have like using significant figures or converting units. In courses in your Engineering Physics curriculum it will be assumed that you have these fundamental skills described in this document, but very little mention will be made of these fundamentals unless you make mistakes and sometimes not even then. The point is that these skills may get overlooked by you for the moment, but will inevitably come back to haunt you. Now is as good a time as any to deal with these skills that are often viewed as the picky details. So let's get started.

Engineering Calculations

The first skill that is important is the subject of calculations. This may include order of magnitude approximations (sometimes called *back of the envelope* calculations), using your calculator for a homework problem, or your own computer program to crunch data from research. Each of these calculations are very different and may require very different resources, precision, and total time to complete. Although it is an important skill to be able to differentiate the requirements to perform a calculation, this is not the subject covered by this document. This document is instead intended as an introduction to the techniques you should use to perform the calculations. It is also intended to serve as a reference for you when confronted with problems in more advanced courses.

Engineers and scientists make many calculations and estimates and base decisions on these calculated or estimated values. Often these skills are overlooked as trivial, but are probably the most common source of errors even for experienced engineers. As an example the rules determining the proper number of significant digits for a measured value are really not that difficult to follow, but if these rules are either not known or are forgotten then errors may be made. Even if these errors are small (i.e. one digit) the consequences may still be significant since most values are used for subsequent calculations. Another reason a single digit error may be important is that the one digit may mean additional costs to your company or a client.

Scalar values

This part of the document is focused on numerical calculations so the starting point should be numbers – in particular we should deal with *scalars*. Scalars are pure numbers like *5* and as such represent only a magnitude. Many quantities in physics and engineering are scalars such as temperature, distance, or energy – so it must be important in physics and engineering to be able to deal with scalars. There are other types of entities in physics and engineering that are not just magnitudes, but also include directions (vectors) or directional components in other directions (tensors). A brief discussion of vectors is included here, but tensors are left for advanced work in fluid mechanics and other areas.

The basic terminology of scalars is given in Table One.

Table 1 Mathematical nomenclature for scalars.

Scalar Type	***Definition***	***Comments***
Natural Numbers	The numbers 1, 2, 3….	
Integer Numbers	All Natural Numbers plus 0, -1, -2….	
Rational Numbers	Any scalar that may be written as p/q (q ≠ 0 and p and q both integers).	All rational numbers may be written as a repeating decimal fraction.
Irrational Numbers	Decimal fraction representations never repeat.	Examples: e = 2.71828… = 3.141592…
Real Numbers	Includes all Rational and Irrational that do not include $i^2 = -1$	
Complex Numbers	May be written as $a + bi$	a and b are real numbers, where $i^2 = -1$.

When a scalar represents a physical quantity like temperature, then the scalar will have units – in this case degrees Celsius would work. Units will be discussed more thoroughly in a subsequent section.

To wrap up this initial discussion of scalars several comments are in order. First, it will be assumed that arithmetic operations with scalars are familiar to you and that evaluating a more involved string of arithmetic operations is an easy process for you. So the following should be a simple evaluation $(3^2 \times 2 + 4)/11 = 2$. Second, this document for the moment deals only with decimal or base-10 scalars. Number systems with other bases are certainly of use, but are left to other courses. Finally, you should be able to perform the scalar operations one a series of one-digit scalars without the use of a calculator. It turns out estimating the result of a somewhat complicated calculation can be an efficient way to check a calculator result (calculator errors are common). This process of estimating the result of a complicated equation amounts to performing simple arithmetic operations on single digit scalar values. This proficiency with single digit scalars should include dealing with non-decimal values i.e. fractions.

Scientific and Engineering Notation

Since many scalar values are either very large or very small (relative to a single digit) a convenient way to express scalar values is to use scientific notation. You may already be familiar with *scientific notation*, but this section will serve as a brief review. As an example let's say we wanted to write the length 123 cm in scientific notation. The result would look like

$$1.23 \times 10^2 \text{ cm}$$

Of course 10^2 is the same as 100, so multiplying the value 1.23 times 100 gives the correct number of centimeters. Note, it is standard to write numbers in scientific notation in the form

$$m.nop\ldots \times 10^q$$

There is one digit before the decimal and the remaining significant digits (discussed thoroughly in a subsequent section) are placed after the decimal. The exponent *q* may be any integer (as described in a previous section). As another example note that 123 cm is the same as 0.00123 km. So in scientific notation this would be

$$1.23 \times 10^{-3} \text{ km}$$

Engineering notation is similar to scientific notation, but different in that the exponent must always be a multiple of three (3) (this includes zero). This rule for the exponent means that many numbers may not be able to be written in the format of scientific notation where one writes only one digit before the decimal and the remaining digits after. Instead, in engineering notation, the first number is always written between one (1) and 1000 to accommodate writing the exponent as a multiple of three. For example if a radiation counter registers 15,255 counts in a one minute period, then the resulting count rate in engineering notation would be

$$15.255 \times 10^{3} \text{ counts / minute.}$$

Most likely your calculator can be set up to display in either scientific or engineering notation. If I enter the above radiation counter measurement in my Casio™ *fx-115W* the display appears as

$$1.52550000^{\ 04}$$

when the calculator is set to display in scientific notation. When the calculator is changed to engineering notation the display shows

$$15.255^{\ K}$$

The superscript suffix *K* means times 1000 on my calculator. If a significantly more sensitive detector is used and a measurement of 15,255,000 counts are observed in some time period, then the value entered on my calculator would appear as

Scientific: $1.5255^{\ 07}$

Engineering: $15.255^{\ M}$

Here the superscript suffix *M* means x 1,000,000. The value entered as 0.000325 will display on my Casio™ as:

Scientific: $3.25^{\ -04}$

Engineering: $325^{\ \mu}$

here the Greek letter µ means times 10^{-6}. The symbols used are part of a convention used with certain physical units and will be explained in more detail in the section on units.

There is another way of displaying scientific notation that is very common on calculators and in computer software. In this notation the above example of 0.000325 would be displayed as

Scientific:	3.25E-04
Engineering:	325E-06

where the E refers to exponential notation. So the E just means times 10 raised to the power of integer following the E. Note it is unlikely that your calculator displays scientific and engineering notation the same as mine.

Scientific/Graphing Calculators

For most of your courses in engineering and physics you will need a scientific or graphing calculator. The available of features and capabilities available for calculators today is very robust. Luckily, many low-priced scientific calculators have 99% of the capabilities you will need. It is OK to have a feature-heavy calculator capable of performing symbolic integration, graphing, and other tasks, but in my experience most students do not know how to use most of these features. For this reason even if a high-powered calculator could help with a difficult problem (say on an exam) most students do not know how to use these calculators to solve those problems. So for my money it is best to go with a scientific calculator under $30 that is typically easier to use (and oftentimes faster).

Even the low-end calculators are capable of doing quite a lot, but many simple calculations require knowing the meaning of some arcane symbols printed on the face of your calculator. For example to generate the calculator displays that are shown in the previous section I had to change the mode of the display back and forth several times. The ability to change this display is critical for every calculator user, but differs among calculator models. The primary way that one learns how to change the calculator mode and use calculator features is to **HAVE ACCESS TO THE MANUAL THAT CAME WITH YOUR CALCULATOR.** I know it is very possible that you do not know where the manual is, but you will definitely need the manual during your academic career. Many of you have very sophisticated calculators with advanced graphing, symbolic, computational, and storage capabilities. It is critical that you obtain a copy of the manual for your calculator. Most calculator manufacturers publish these manuals on their web-sites. For your reference here are links to web locations where you may find manuals:

Casio Manual Page:
http://www.casio.com/
Texas Instruments (navigate to your specific product to find your Guidebook):
http://education.ti.com/us/
Sharp Electronics (not that many manuals available at this printing):
http://www.sharp-usa.com/
Hewlett-Packard (Pretty hard to find the manuals here):
http://www.hp.com/

Significant digits

*A **significant digit** is any scalar is any number that may be specified with confidence.* As an example consider the number *9302.02* where all digits are significant (this of course would only be known initially by the person stating this value). This number has *6* significant digits – The *9* on the far left is the **most significant digit** and the *2* on the far right is the **least significant digit**. There are several ways one may know which digit is the least significant digit, and knowing in which digit you are least confident (but still confident) depends on how the number was gotten (e.g. a measurement versus a calculation). These specific techniques for determining which digit is least significant are not discussed for the moment, but assume that you know which digits are significant and you are ready to report your value. The only thing you need to know to report this value is the *Rule for Writing Numbers*, which says

> **Rule for Writing Numbers**
>
> Any nonzero digit in a number is a significant digit; and the digit zero (Ø) i s significant digit unless it is used to specify the location of the decimal place or used to fill the location of discarded digits.

If numbers are written in the manner specified by the *Rule for Writing Numbers* by someone who knows which digits are truly significant, then one can determine the number of significant digits as follows:

> **Counting Significant Digits**
>
> Count the number of digits beginning with the first nonzero digit on the left and ending with the last digit on the right - unless that digit is a zero (Ø) and satisfies one of the exceptions in the *Rule for Writing Numbers* above.

Very often it is not quite clear what the person that wrote a number intended. For example, if someone tells you that they measured a table and found it to be *320 centimeters* long, then does that length have two or three significant digits? You might guess it has three, but it actually depends on how the measurement was made. One way to alleviate the confusion would be to specify the decimal point as *320. centimeters*. This implies that the zero is **not** used to specify the location of the decimal point, so that the number has three (3) significant digits. If the decimal point is missing, then one would have to assume that the number only had two (2) significant digits.

Another way to alleviate confusion is to write numbers in **scientific notation**. So that table length would be 3.20×10^2 *centimeters*, which clearly has three (3) significant digits, while 3.2×10^2 *centimeters* has only two (2) significant digits.

Another common technique used in textbooks is to only use zeros at the end of a number if they are significant. For example if a problem specifies that a wheel has a radius of *2.00 meters* the author intends for you to consider the radius to have three (3) significant digits. Often textbooks will inform you in the first few sections that you should assume that numbers specified in textbook problems always should be considered to have three or four digits or the professor teaching the course may inform you of his/her preferences. You should carefully read your text to make sure you understand what is intended.

Rounding

Rounding significant digits is another potentially confusing issue that arises due to the fact that one carries **extra** figures (either by hand or stored in a calculator or spreadsheet) during intermediate calculations. For example despite the fact that your calculator might show up to 13 digits in a result, not all of those digits are warranted. Assuming the number of significant digits is known one should use the rules for rounding that are summarized in Table Two.

Table 2 The rules for rounding numbers with a given number of significant digits.

When the first digit dropped is:	The last digit retained:
< 5	Stays the same
>5 or a 5 followed by any nonzero digits	Is increased by 1
5 not followed by nonzero digits	Stays the same if even Is increased by 1 if odd

This rounding rule may be a little different than what you have been told or used in the past, but if followed it tends to minimize errors due to rounding.

As an example of using the rounding rule from Table Two consider that the number *3.1545* appears on your calculator. If one rounds this number to one (1) digit, the result would be *3*. Rounding this value to successfully larger numbers of significant digits yields the results in Table Three.

Table 3 Example of rounding the number *3.1545*.

Number of Significant Figures	Rounded Number
2	3.2
3	3.15
4	3.154

The last row of the table may surprise you, but the rules for rounding in Table Two indicate that an even number stays the same when it is followed by a five (5) and nothing else – this rule minimizes rounding errors when followed consistently. The results of rounding the number *3.1575* are shown in Table Four.

Table 4 Example of rounding the number *3.1575*.

Number of Significant Figures	Rounded Number
2	3.2
3	3.16
4	3.158

Please note that the last row in Table Two does occur, although certainly less frequently than the cases shown in the previous rows.

Another important skill in numerical calculations is arithmetically combining numbers with differing numbers of significant digits. There are very general rules to determine the number of significant digits to retain after any given mathematical operation, but these rules will be left to more advanced courses in your curriculum. For now the rules for addition/subtraction and multiplication/division will be stated. Note that these rules will get you quite a long way.

Addition/Subtraction. In the process of adding or subtracting two numbers the result should have no digits beyond the last estimated decimal place of the number that had the fewest number of decimal places. Despite the confusing sound of this rule, it's pretty simple. Consider adding two numbers: *3.2* and *7*. If one writes these numbers one above the other as follows

```
  3.2
+   7
---------
 10        (both digits are significant)
```

The number *7* limits what we can do since it has zero decimal places. We still end up with a result with two (2) significant figures, but we cannot use the *2* (of the *3.2*). We may however have to use the *2* to know how we round the result. For instance if the problem had been *3.6 + 7*. The result should be *11*. What would the result be if we tried *3.5 + 7*?

Multiplication/Division. Generally, the rule here is if multiplying/dividing two numbers the result will have the *same or one more* significant digit than the number with the least number of digits that was multiplied/divided. The following examples should make this rule clear (all digits shown are significant):

1.2	x	3	=	3.6		
9.81	/	3.14	=	3.12		
2.34	x	5.1	=	11.9		
2.34×10^4	/	3.1×10^2	=	75	=	7.5×10^1

Practical Engineering Calculations

Engineering data is seldom known to better than 0.2%. As an example consider the relative error between *9.97* and *9.99* (about 0.2%) and between *1.001* and *1.003* (about 0.2%) . Unless otherwise specified it is best to report final results to engineering computations to either three or four significant digits. Here is the rule of thumb:

Engineering Calculation Rule of Thumb

When using typical engineering data carry through intermediate calculations with more digits than justified (to minimize rounding error) and report the final results with four (4) significant digits if the first digit is a *1*; otherwise report the result with three (3) digits.

Engineering Units

SI or metric units are used fairly often in engineering calculations, but frequently *English Engineering Units* are used. This section is presented to help you properly use these engineering units. All units systems have a set of base of fundamental quantities from which almost all other units mat be derived. Usually the derivation occurs by using a constitutive relationship like Newton's Second Law. Table Five shows which units are fundamental and which are derived for several units systems.

Table 5 Fundamental and derived units for several unit systems.

Dimension	British Gravitational	SI	English Engineering
Mass (Fundamental)		kg	lb_m
Force (Fundamental)	lb_f		lb_f
Mass (Derived)	slug		
Force (Derived)		N	
Length	ft	m	ft
Time	s	s	s

Since English Engineering units have mass and force each as fundamental a factor is required to make sure mass and force have the correct proportionality. This is achieved using applying Newton's Second Law and by defining one lb_m as the mass that creates a weight of one lb_f in a gravitational field with acceleration of 32.174 ft / s^2. Newton's Second Law may be written as

$$F = kma$$

Equation 1

Where *F* is the force that would create an acceleration of magnitude *a* for a mass of *m*. the factor *k* is the proportionality factor to make sure we get the correct force for a given mass. For all unit systems besides the English Engineering system, *k* has a value of one (1). For English Engineering *k* will have the following value

$$k = \frac{1}{32.174\dfrac{lb_m - ft}{lb_f - s^2}}$$

Defining the g_c as follows

$$g_c = 32.174\frac{lb_m - ft}{lb_f - s^2}$$

Consider the example of an object with a mass of 10 lb_m. What would the weight of this object be in a gravitational acceleration of 32.174 ft / s^2? Using Eq.(5) one obtains

$$F = \frac{1}{32.174\dfrac{lb_m - ft}{lb_f - s^2}}(10lb_m)(32.174\,ft/s^2) = 10lb_f$$

So since the acceleration has the numerical value of 32.174 ft / s^2, the weight has the same numerical value as the mass.

In engineering, frequently you will need to convert the units of a quantity. Unit conversion is relatively simple. The two examples below show how units may be converted by multiplying by the appropriate conversion factors

$$25\frac{m}{s}\times\frac{3.281\,ft}{m}\times\frac{1mi}{5280\,ft}\times\frac{3600s}{hr} = 56\frac{mi}{hr}$$

$$1.03\frac{slug}{ft^3}\times\frac{14.594kg}{slug}\times\left(\frac{3.281\,ft}{m}\right)^3 = 531\frac{kg}{m^3}$$

Most physics and engineering textbooks include tables of conversion factors in an appendix or elsewhere. You might get familiar with a particular conversion table in one of your textbooks. When you are in front of a computer you can always use http://www.onlineconversion.com/, which will perform many conversions.

Dimensional Analysis

Physics and Engineering Equations must be dimensionally consistent – that is each side and each additive term must have the same dimensions. Dimensions are just mass, length, time, etc. Consider the equation for position versus time with uniform acceleration in one-dimension

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

Now let's examine the dimensions of each quantity

$$[L] = [L] + [\frac{L}{T}][T] + [\frac{L}{T^2}][T^2]$$

where dimensions of length, *L*, and time, *T*, are represented in square brackets. Clearly, each additive term in the equation has the same dimensions – *L*. The fact that physical relationships must satisfy the criterion of being dimensionally homogenous actually allows one to determine the nature of a complicated physical relationship. This process is called dimensional analysis and is a very valuable tool to understand complicated multivariate relationships. There are textbooks completely devoted to this subject, but for the moment let's consider a simple engineering problem and attempt to arrive at a solution using dimensional analysis.

Consider the flow of a fluid over a smooth sphere as shown in Figure One. The basic problem of drag on a smooth sphere has been studied in detail. The variables of interest in this problem are shown in Figure One.

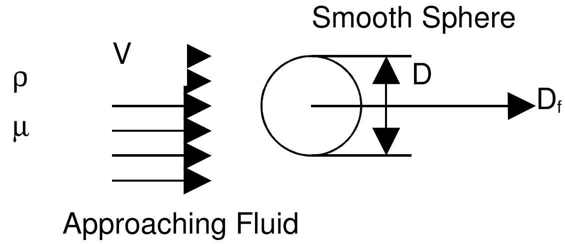

Figure 1 Defining variables for drag due to external fluid flow over a smooth spherical object.

The variables in the above figure have the following meaning:

ρ	=	density of fluid [kg/m^3],
μ	=	absolute viscosity of fluid [N-s/m^2],
V	=	velocity of fluid [m/s],
D	=	diameter of sphere [m], and
D_f	=	drag on sphere [N].

The method of dimensional analysis involves analyzing the dimensions of all of the variables involved and developing dimensionless parameters that describe the problem. Following the Rayleigh Method the drag force D_f is related to the other variables as follows:

$$D_f = A_1 \rho^a V^b \mu^c D^d$$

Equation 2

Where

A_1	=	an arbitrary constant, and
a-d	=	arbitrary exponents.

It may be shown that the functional relationship that arises from Eq.(2) is

$$C_D = f(\mathrm{Re}_D)$$

Equation 3

Where

C_D = the coefficient of drag, and

Re_D = Reynold's number.

$$C_D = \frac{D_f}{1/2 \rho V^2 A}$$

Equation 4

$$\mathrm{Re}_D = \frac{\rho V D}{\mu}$$

Equation 5

Note that in Eq.(4), *A* is the projected area of the object (i.e. area normal to the flow). So in the case of the solid sphere

$$A = \frac{\pi}{4} D^2$$

The Reynold's number has many physical interpretations, but these will be explored later in your curriculum.

The importance of Eq.(3) is twofold: a many-variable problem has been simplified to one independent variable (hence considerably fewer experiments have to be performed to establish a relationship), and experiments may be performed with different fluids (even at different conditions) or different dimensions to allow for modeling and prototyping. These facts are termed *dimensional* and *dynamic similarity*.

Basic Statistics

Frequently in your academic career you will need to know some basic statistics; whether it is to determine if the difference between two measured values is statistically significant or to find your grade point average. In general statistics are just numerical ways of expressing facts concerning data. The primary ways of expressing these facts are through several quantities. To begin with one must assume that you have a "perfect set of data." This data is really imaginary because it requires that an infinite number of measurements be made. One quantity of interest is the **mean, $\bar{x}$,** which is just the **average value** or **centroid** of the numerical data is calculated as

$$\bar{x} = \lim_{N\to\infty}\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

Equation 6

where

N	=	the total number of measurements,
x_i	=	the value of the i^{th} measurement.

Note the summation sign (Σ) has been used to indicate that one would add up each value of the measurement, x_i then divide by the total number of measurements, N to yield the mean. The standard deviation, σ is a measure of how much the data is spread out from the mean and can be calculated as

$$\sigma^2 = \lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2\right]$$

Equation 7

Since in real circumstances one makes a finite number of measurements, Eqs.(1 and 2) have to be modified to the **sample mean** and the **sample standard deviation**, which are defined as

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \ ,$$

Equation 8

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2 .$$

Equation 9

So when one makes another measurement, you could say that most of the time the value will be within σ of $\bar{x}$. To make all of this a little more concrete let's consider the example of measuring the length of a table with a meter stick. Let's say you have repeated the measurement nine times and recorded the lengths shown in Table 1

Table 6 Measured lengths of a table.

Trial #	Length (meters)
1	1.022
2	1.024
3	1.023
4	1.021
5	1.022
6	1.024
7	1.020
8	1.025
9	1.026

The associated statistics for this data and how they are calculated are shown in Table 7.

Table 7 Statistics calculations for table data.

i	x_i (meters)	$\bar{x}$ (Eq. 3) (meters)	$d_i = \bar{x} - x_i$ (meters)	$(d_i)^2$ (meters)^2	σ(Eq. 4) (meters)
1	1.022	1.023	0.001	1E-06	0.001936
2	1.024		-0.001	1E-06	
3	1.023		0.000	0	
4	1.021		0.002	4E-06	
5	1.022		0.001	1E-06	
6	1.024		-0.001	1E-06	
7	1.020		0.003	9E-06	
8	1.025		-0.002	4E-06	
9	1.026		-0.003	9E-06	
Σ	9.207			3E-05	

So the meaning of these statistics is that if you measure the length of the table again (call this measurement x') you will most likely find the length to be

$$\bar{x} - \sigma < x' < \bar{x} + \sigma$$

or

$$1.021 \text{ meters} < x' < 1.025 \text{ meters}.$$

So how do you know how many digits are significant in a given value? There are two ways. First, one may use statistics to determine the **precision** of a given value that has been measured multiple times. The precision just specifies which decimal place contains the last significant digit and the minimum value of that digit. For example if one measures the length of a table with a meter stick several times the result might be stated as

$$1.2550 \pm 0.0005 \text{ meters}$$

where the precision of the measurement is 0.0005 meters. If one has not made multiple measurements, then an estimate of the precision may be made by considering the precision of the device that was used to measure the value. Making multiple measurements is the preferable way to find the precision, but often the second technique is acceptable if multiple measurements (in the range of less than ten measurements) will not dramatically increase the precision.

In the case where you made a measurement of a value or you know the details of a measurement, then determining the number of significant digits is a matter of statistics. For example if you are counting objects (let’s say the number of cars in a parking lot) and you count *128* cars in a parking lot, then how many significant digits does *128* have? The answer is probably three (3), but it may depend on how carefully you counted or if you checked yourself by counting multiple times. Let’s say you counted the cars five times and came up with the numbers shown below in Table 4.

Table 8 Car counting results.

Trial #	# of cars
1	128
2	125
3	128
4	129
5	130

How many cars are in the parking lot? Well now one has to do a little statistics to find out. The average number of cars using Eq.(3) is 128, but is *128* three (3) significant digits? To know one must know the standard deviation for the calculation. As calculated using Eq.(4) the standard deviation is *2*. Note that the **standard deviation is only specified with one digit**, and that digit must be able to be added to or subtracted from *128*. So the proper answer to the question how many cars are in the parking lot? is 128 ± 2. The last digit shown (i.e. the *2*) is significant, since we know that there is a good chance (around 68% in fact) that if we do the counting many times that the average will fall between 126 and 130. So the last digit is important and may be specified with confidence; thus 128 ± 2 has three (3) significant digits. You will learn the specifics of how to estimate the precision without making multiple measurements when you take *PHY 2014L Physics for Scientists and Engineers I Laboratory*.