Intro to Engineering Spring 2006
Falling Sphere Viscometer Competition
Competition will take place on Thursday May 4, 2006. Presentations from 11:00 -11:50 a.m. and competition from noon-12:50 p.m.

Introduction

This semester your team will be asked to predict the fall time for a sphere through given heights of 2 liquid columns. You will know the dimensions of the device through which the sphere will fall, the properties of the sphere, and the properties of the liquid. These quantities along with information (that will be provided) about drag forces on spheres will allow your team to predict the fall-time of the spheres. Also you will make a prediction of the liquid viscosity.

The drop tubes are made of acrylic tubes and plexiglas bases. The *maximum height of the drop* will be 25 cm. The insider diameter of the acrylic tube you will use is 4.47 cm.

Design Teams are the same as the Robolab competition.

Nature of Viscosity

Viscosity causes internal fluid friction. It also makes fluids stick to solid surfaces. Shear forces are transmitted through a fluid by it viscosity. This can be seen if an experiment is performed as shown in Figure 1.

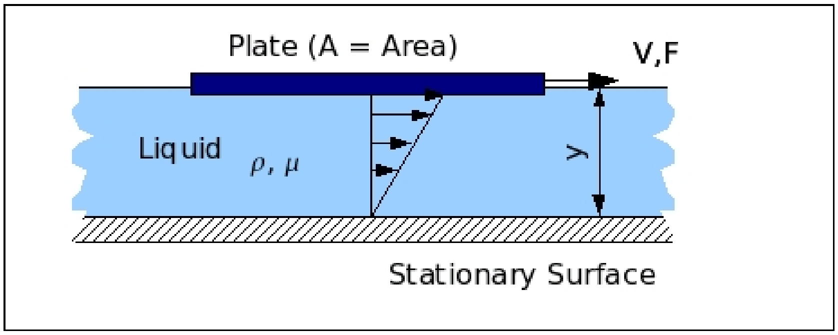

Figure 1: A plate is dragged across the surface of a fluid. This creates shear stresses in the fluid.

The viscosity causes the liquid to stick to the stationary surface on the bottom and to the plate on the top – so the fluid at the bottom has a velocity of zero and the fluid at the top has a velocity of V (i.e. the same as the plate). The fluid in the middle is sheared by the plate on top and the stationary surface on the bottom. The viscosity can be defined in terms of Newton's Law of Viscosity:

$$\tau = \frac{F}{A} = \mu \frac{V}{y}$$

Equation 1

Variable	Definition	Units
τ	Shear stress in fluid	Pa
F	force on the plate	N
A	area of the plate (one side)	m^2

Variable	***Definition***	***Units***
μ	viscosity	kg/m/s
V	velocity of the plate	m/s
y	distance from bottom surface to plate	m
ρ	density of fluid	kg/m^3

Although this experiment may be done in practice it is more common to use a rotating cylinder or cone with a known applied torque which once the the angular velocity is measured may be related to viscosity. There are many other ways to measure viscosity – the next section discusses the falling sphere viscometer, which is what you will be using for your project in this course.

Drag Forces on Spheres

Figure 2 shows the arrangement for the falling sphere viscometer. The length from which the sphere is dropped is L_{cyl} while the length where the sphere is speeding up to terminal velocity is L_0 and the length over which the sphere is traveling at terminal velocity is L_t. Given the setup in Figure 2 your team's job is to predict the total time for the sphere dropped from a height L_{cyl} to reach the bottom of the cylinder.

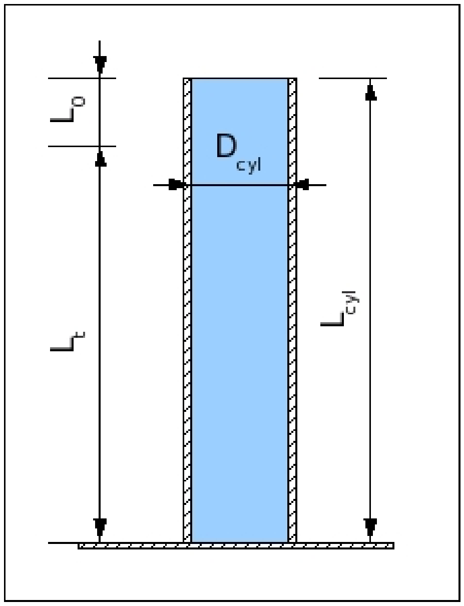

Figure 2: The falling sphere viscometer.

An analysis is given here for the sphere falling at terminal velocity. Figure 3 shows a free body diagram of the sphere. In Figure 3 F_D is the drag force exerted by the fluid as the sphere drops downward, F_W is the weight of the sphere, and F_B is the buoyancy of the sphere.

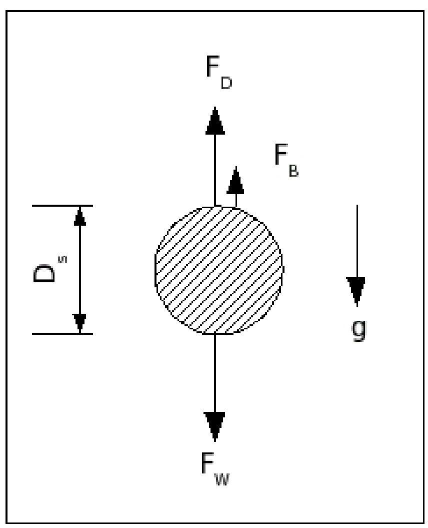

Figure 3: Free body diagram of a sphere falling through a fluid.

$$F_W = mg = \frac{\pi}{6} D_s^{\,3} \rho_s g$$
$$F_D = C_D \frac{1}{2} \rho V^2 \left(\frac{\pi}{4} D_s^{\,2}\right)$$
$$F_B = \frac{\pi}{6} D_s^{\,3} \rho g$$

Equation 2

Note that C_D is the coefficient of drag and may be found on a graph that you will be given. To use the graph you will also need $\Re_D$ (Reynold's number)

$$\Re_D = \frac{\rho V D_s}{\mu}$$

Equation 3:

Note that diameters and properties like density and viscosity that do not have subscripts are for the liquid.

If the sphere is falling at constant speed (terminal velocity) then Newton's second law applied to Figure 3 gives

$$F_D + F_B = F_W$$

Equation 4:

Eq.(4) along with Eqs. (2) and (3) in conjunction with the graph may be used to determine the terminal velocity V_T. If the sphere is falling with terminal velocity then for the distance L_T the time to fall this distance will be

$$t_T = \frac{L_T}{V_T}$$

Equation 5:

Terminal Velocity Calculations

One thing you will have to do is understand how to to find the terminal velocity for a particular sphere falling through a particular liquid. This may be done in conjunction with the graph of C_D versus $\Re_D$ that you were given. Let's do an example:

Example 1: Terminal Velocity Calculation

Find the terminal velocity of a steel ball bearing (ρ = 7.80 x 10^3 kg/m^3) with a diameter of 1.00 cm in SAE 10 oil at 20°C (ρ = 860. kg/m^3 and μ = 0.100 kg/(m-s)).

First of all when the sphere is at terminal velocity the drag force plus the buoyant force equals the weight of the sphere. The weight of the sphere and the buoyant forces are

$$F_W = mg = \frac{\pi}{6} D_s^3 \rho_s g = \frac{\pi}{6}(0.0100\, m)^3 x\, 7.80 x 10^3\, kg/m^3\, x\, 9.81\, m/s^2 = 4.01 x 10^{-2}\, Newtons$$

$$F_B = \frac{\pi}{6} D_s^3 \rho g = \frac{\pi}{6}(0.0100 \mathrm{m})^3\, x\, 860 \mathrm{kg}/m^3\, x\, 9.81\, m/s^2 = 4.42 x 10^{-3}\, Newtons$$

Combining F_W and F_B to get F_D from Eq.(4) gives

$$F_D = F_W - F_B = 3.57 x 10^{-2}\, Newtons = C_D \frac{1}{2}(860.\, kg/m^3)\, V_T^2 (\frac{\pi}{4}(0.0100 \mathrm{m})^2)$$

Solving for $C_D V^2$ gives

$$C_D V_T^2 = 1.06\, m^2/s^2$$

Now let's get $\Re_D$ as far as we can

$$\Re_D = \frac{\rho\, V\, D_s}{\mu} = \frac{(860.\, kg/m^3)(0.0100\, m)}{0.100\, kg/(m-s)} V_T = (8.60\, s/m)\, V_T$$

We cannot go any further without the plot of drag coefficient versus Reynold's number. But with that graph we can guess values of V_T then calculate $\Re_D$. This allows us to look up a value of C_D. Then we know that the combination C_D V_T^2 = 1.06. If the guessed value of V does not match up with this value then we need to re-guess V_T . It is left as an exercise to find the appropriate value of V_T.